

# Teaching in Context: Equivalence in the Elementary School

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AERA Annual Conference in New Orléans 2011

# Mathematical Relations

- Is the same shape/colour/size/quantity/length/time/weight/capacity as
- Has the same value/area as
- Is similar to
- Is congruent to
- Is parallel to
- Is even and so is
- Is less than
- Is greater than
- Is perpendicular to
- Is not greater than
- Is divisible by
- Is taller than
- Is longer/shorter than
- Represents the same number as
- Can be exchanged for
- Speaks the same language as/is married to/is a sister of/is an uncle of

# Mathematical Topic: Equivalence

- Equality
- Mathematical relation, with three properties
  - $a \sim a$  (symmetry)
  - If  $a \sim b$ , then  $b \sim a$
  - If  $a \sim b$  and  $b \sim c$ , then  $b \sim c$
- Important for algebra
- Opportunity for students to reason

# Question of Interest

How were a teacher's knowledge, beliefs and goals evident in decisions made during a teaching episode from a series of lessons taught during a ten-hour mathematics summer school?

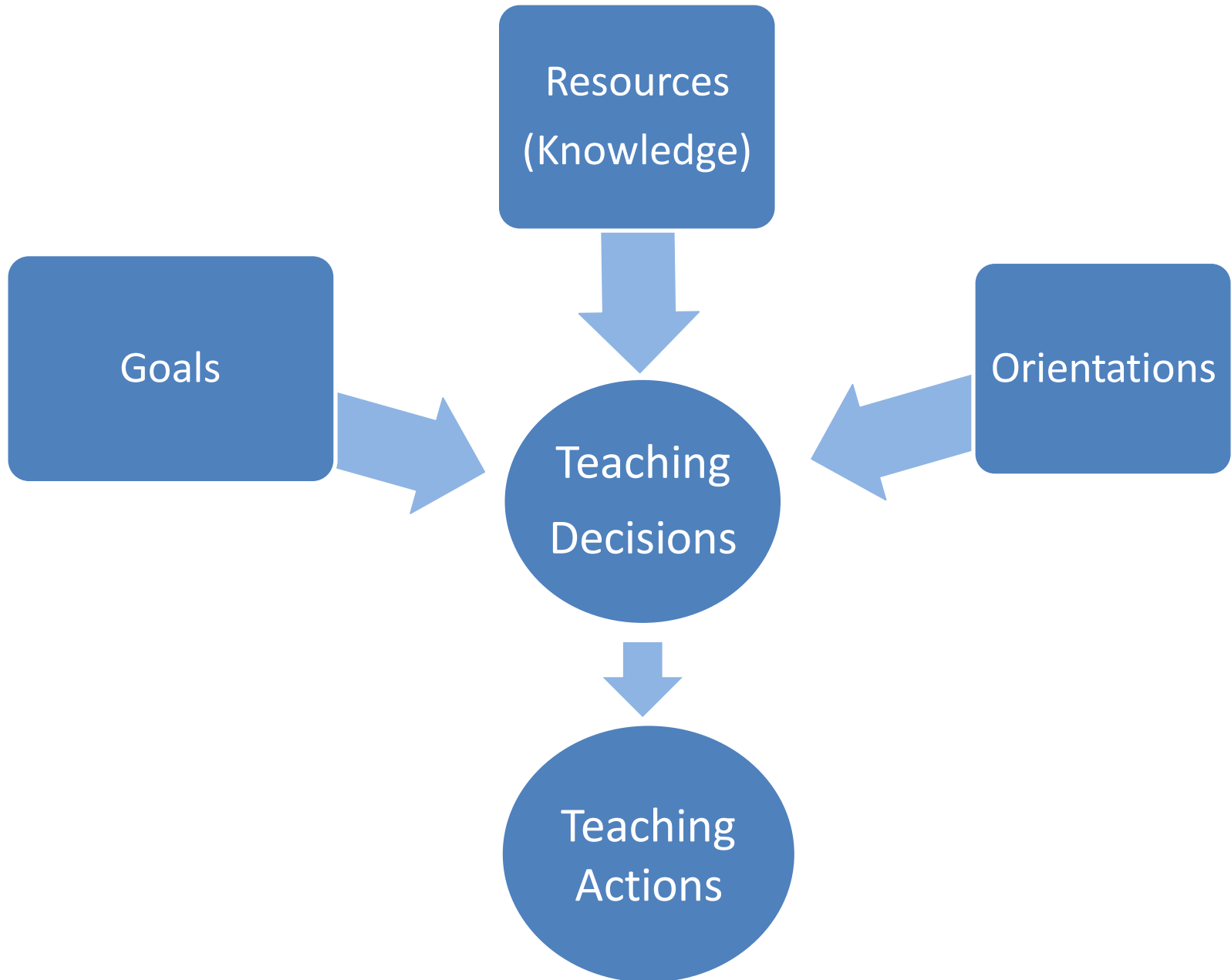
# Context

- One week, July 2009
- 10-hour summer school for 3<sup>rd</sup> grade children
- 20-hour summer course for teachers
- 26 students: 15 boys, 11 Girls
- Students recruited from 11 Schools
- Students' parents at least are motivated to learn mathematics
- No records from previous teacher
- Teacher specialised in mathematics education
- Not using a textbook
- Mathematics laboratory

Laboratory Class Pictures Removed

# Method

- Collected video records and transcribed the discussions
- Also collected teacher's lesson plans, classroom artefacts and children's work
- Using Schoenfeld's "teaching in context" to analyze "action sequences" from the class



Drawing based on Schoenfeld (1998, 2011)



# Excerpt

- Are these number sentences true or false
- $12 - 9 = 3$
- $34 - 19 = 15$
- $43 = 27 + 16$
- $5 + 7 = 11$
- $58 + 76 = 354$
- $27 + 48 - 48 = 27$
- $345 + 568 - 568 = 353$
- $48 + 63 - 62 = 49$

All taken from Carpenter, Franke & Levi, 2003

# Standard Routine

Number sentence on board;  
student asked if true or false



Justification of response  
sought and received

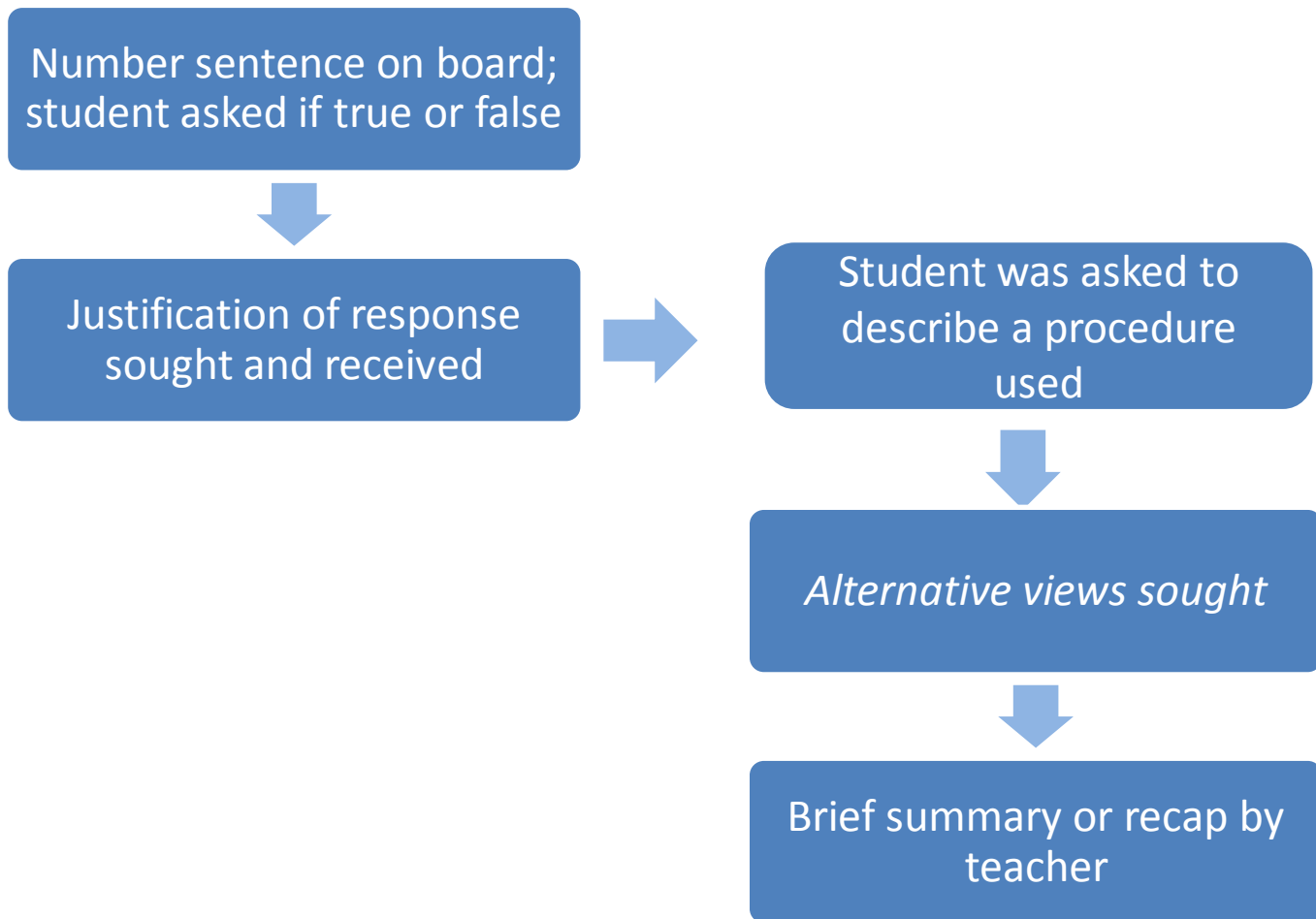


*Alternative views sought*

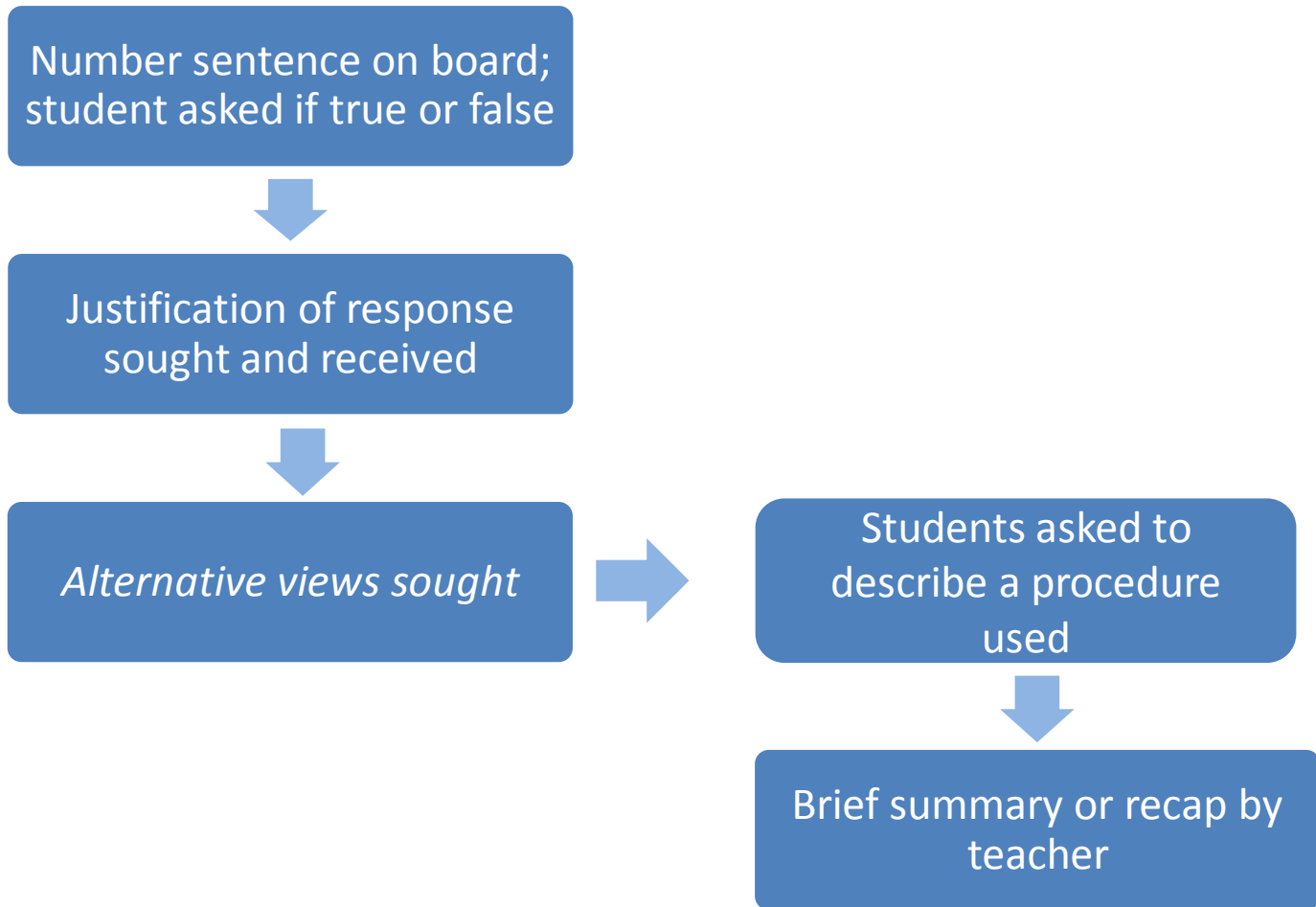


Brief summary or recap by  
teacher

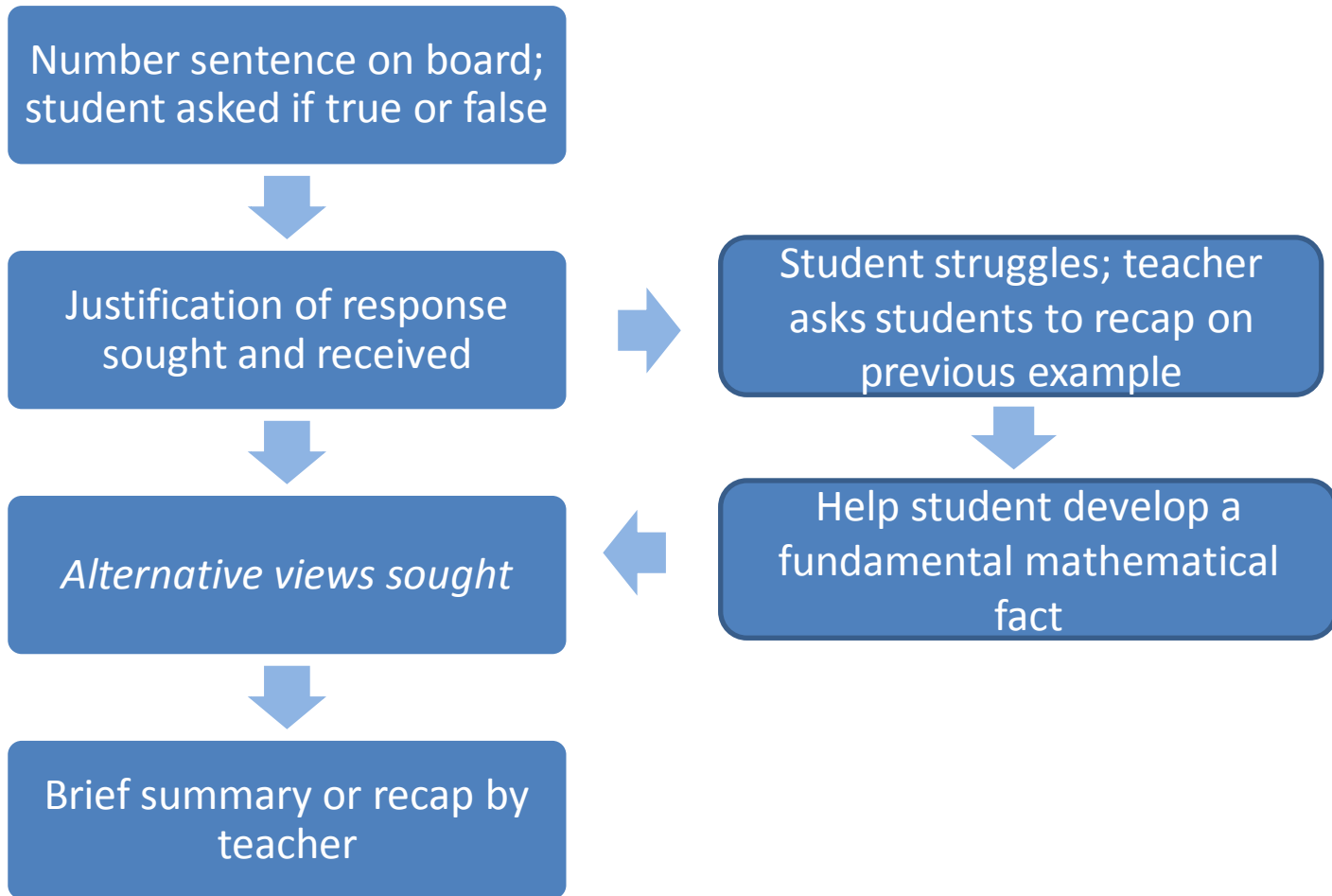
# Standard Routine: Variation 1



# Standard Routine: Variation 2



# Standard Routine: Variation 3



# Goals

## **Mathematical**

- Have students evaluate number sentences as true or false and justify their judgment
- Highlight a student's contribution that is related to a key lesson goal or that is central to mathematics more generally
- Seek comment and responses to student contributions from other students
- Ask students to reword contribution of another student
- Identify connections between children's methods
- Elicit a description of a procedure used
- Respond to a student's comment
- Provide support to a student who is stuck

## **Classroom Management & Establishing Classroom Norms**

- Ensure that teacher and all students hear each student who addresses the class
- Affirm constructive classroom behaviour

# Orientations

## Toward learning

- L1. Students can learn from one another and can use one another's ideas
- L2. Students can learn from hearing others describe procedures used
- L3. Students learn from paraphrasing other students' explanations

## Toward teaching

- T1. Students' ideas need to be probed
- T2. Some ideas from students need to be highlighted
- T3. Some ideas need to be challenged

## Toward Students

- S1. All have important ideas to contribute

## Toward Classroom Environments

- C1. All students need to be able to hear one another and all students are worth hearing
- C2. Students need to give one another time to think, even if they know the answer

## Toward Mathematics

- M1. Statements, whether true or false, require explanation or justification
- M2. Some ideas are worth pursuing even if they are not part of the focus of the lesson
- M3. More than one explanation or justification of a response is possible
- M4. A problem can be solved by referring to a problem already solved

# Interaction of Goals, Orientations & Resources (Knowledge)

- Goal: Have students evaluate number sentences as true or false and justify their judgment v. Orientation: Some ideas are worth pursuing even if they are not part of the focus of the lesson
- Goal: Highlight a student's contribution that is related to a key lesson goal v. knowledge of identifying the contribution when it is made
- Goal: Provide support to a student who is stuck v. Knowledge of how to assess understanding of topic



# Implications

- Pre-service and in-service mathematics education for teachers needs to attend more to developing awareness of how beliefs and knowledge and goals interact
- Teaching the topic of equivalence has potential to promote student reasoning
- Studying how this occurs in practice of a school setting is needed
- Further analysis needed to understand the interaction as it occurs in practice