

Teaching Mathematics to Promote Children's Thinking

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Introduction 1

- Who teaches senior classes (3rd to 6th)?
- Who teaches junior classes (J.I. To 2nd)?
- Who teaches learning support?
- Who teaches in a special school or class?
- Who is a teaching principal?
- Who is a mostly administrative principal?

Introduction 2

- What would you like to work on?
- What I say won't apply **exactly** to your classroom situation
- You won't be leaving here with resources that you can use in class tomorrow!
- Share some ideas and experiences about teaching maths

Patterns of maths lessons

A

Review homework or remind children of accomplishments so far

Present the topic and problems for the day

Students or teachers develop procedures to solve the problem(s) at the board, taking suggestions from other students and the teacher

Practise doing problems similar to those worked on above

GERMANY

B

Review the previous lesson with a brief teacher lecture or summary by students

Present the problem for the day. One problem only.

Students work individually on problems first and then in groups for 5-10 minutes

Students (selected by teacher) present and discuss one or more solution methods.

Highlight and summarise major points

JAPAN

www.seandelaney.com

C

Review previous material: Check homework or engage in warm-up activity

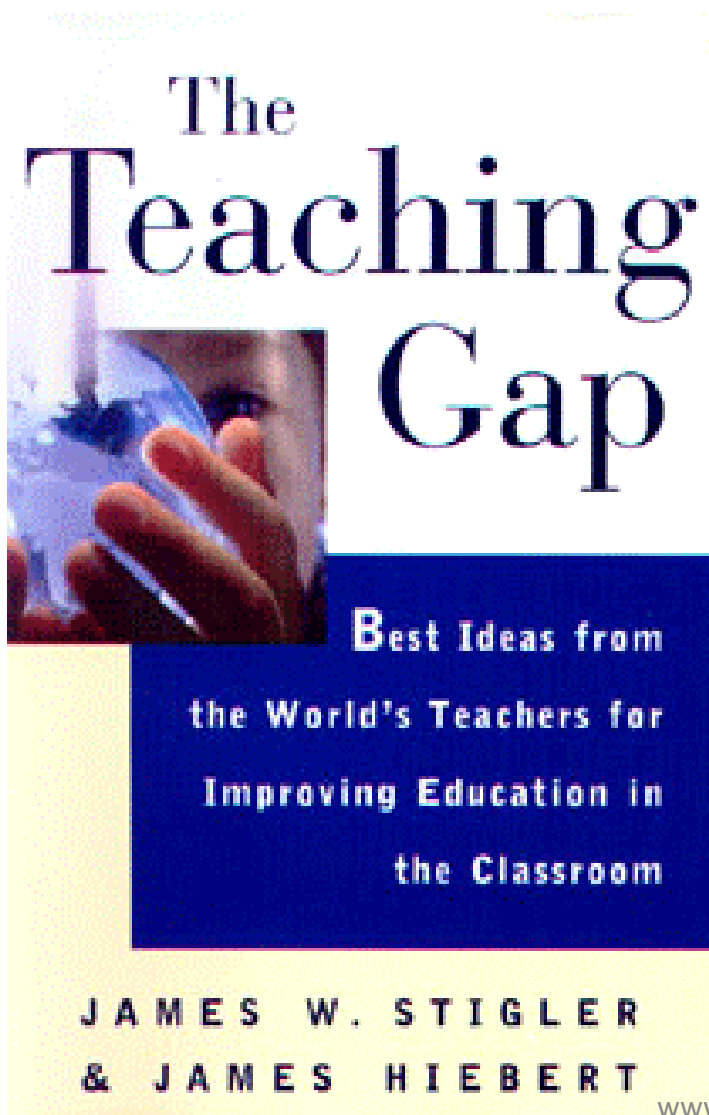
Demonstrate how to solve problems for the day by presenting a few sample problems and demonstrating how to solve them

Practise problems similar to those for which solutions were demonstrated

Correct class work and assign homework

UNITED STATES

“Teaching is a Cultural Activity”



“Some people think that teaching is an innate skill, something you’re born with. Others think that teachers learn to teach by enrolling in college teacher-training programmes. Neither is the best description. Teaching, like other cultural activities is learning through informal participation over long periods of time. It is something one learns to do more by growing up in a culture than by studying it formally.”

“If you always do what you’ve always done,
you’ll always get what you’ve always got.”

If what you’re doing isn’t working, try something
new.

Mathematical Knowledge for Teaching

$$\begin{array}{r} 35 \\ \times 25 \\ \hline 875 \end{array}$$

How Did this Student Get this Answer?

$$\begin{array}{r} 35 \\ \times 25 \\ \hline 255 \\ + 800 \\ \hline 1055 \end{array}$$

Example from Deborah Ball

Clip 1: Mathematics Question

How much would $\frac{1}{4}$ kg of mushrooms cost if the price was €0.62 per 100g?



Image taken from <http://whatscookingamerica.net/Mushrooms.jpg>

Clip 2: Define an Odd Number



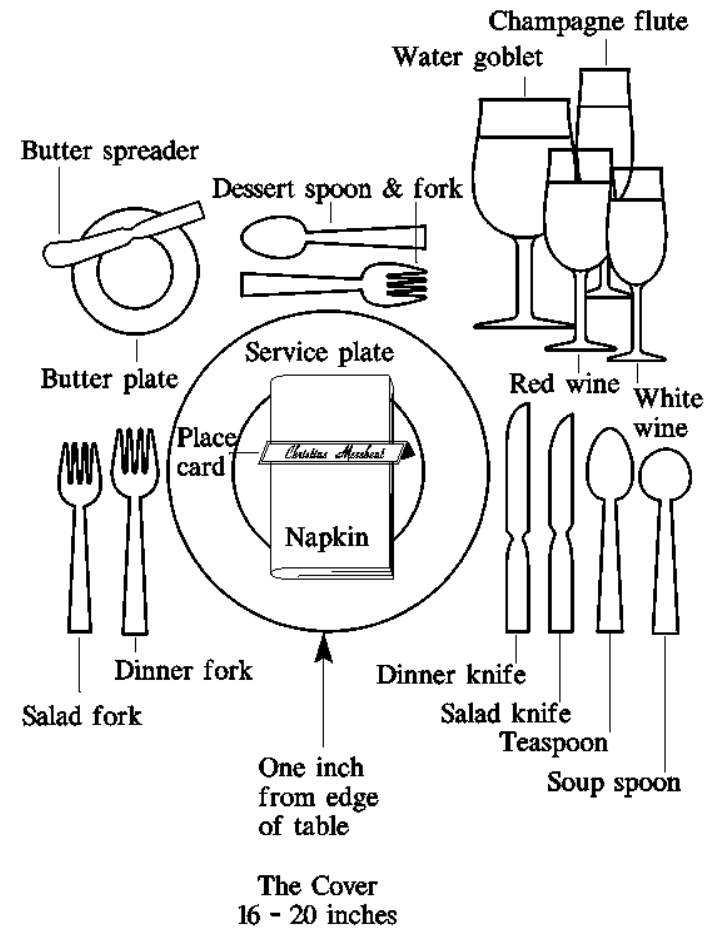
Piaget's Theory of Mathematical Knowledge

- Physical knowledge
- Social Conventional Knowledge
- Logico-Mathematical Knowledge

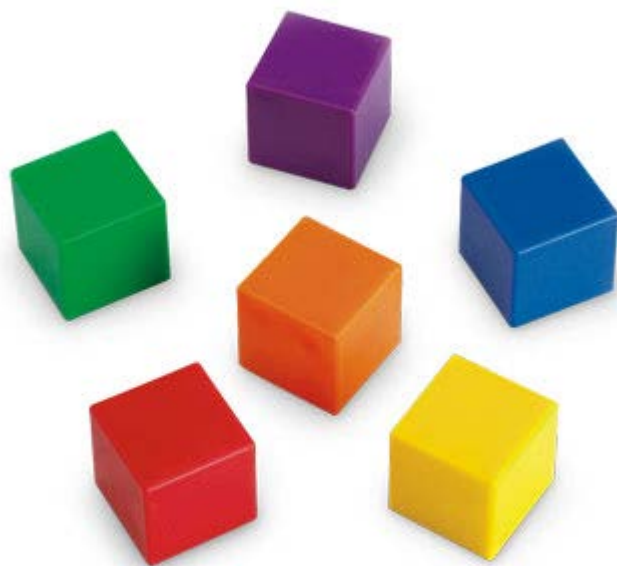
Physical Knowledge



Social Conventional Knowledge



Logico-Mathematical Knowledge



Links to Kamii Videos

<http://www.youtube.com/watch?v=MCaAeGcZF>
WI

<http://www.youtube.com/watch?v=cz4t51wBGI>
o

<http://www.youtube.com/watch?v=->
HEUgHlco w

The key to improving mathematics teaching is...

...NOT



BUT...

...getting children to think and talk
mathematically

Asking questions such as...

Why?

HOW DID YOU GET THAT?

Estimate

Could it be done
another way?

EXPLAIN

Compare

Tell me about...

Put into your own words

How does that connect with...?

Describe

How can you be
sure?

Do you agree or disagree?

Asking questions such as...

Why?

HOW DID YOU GET THAT?

Estimate

Could it be?

Why?

EXPLAIN

Tell me about...

Compare

Put into your own words

What does that connect with...?

Do you agree or disagree?

How can you be sure?

...and genuinely engaging with the response.

Help Children Develop their Mathematical Language

- Factor
- Odd
- Share
- Difference
- Product
- Sum
- Prime
- Face
- Improper ...

And Children can **Write** about Maths

Prompts

- Write down everything you know about...
- Write down what you learned in class today
- Write down one question you still have about...
- After our discussion, write down what you think...
- What would you say to some who thinks that ...?
- Write down what you would like to learn about in maths next week

Some Samples of Children's Writing about Maths

I learned what quotient means
this week.

Next week I would like to learn more
about remainders

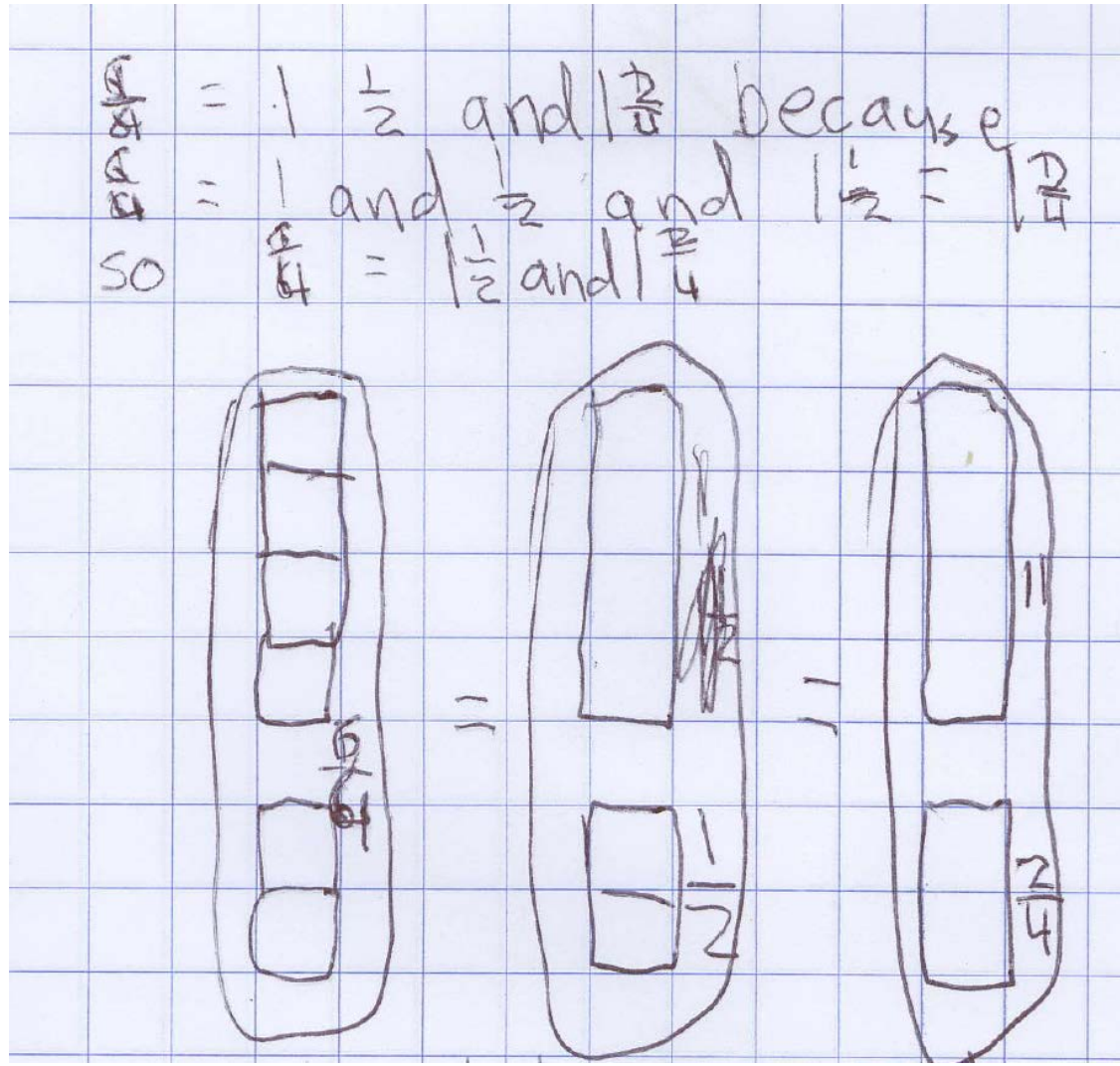
I would like to learn about ~~time~~ time
I learned about solving problems.

Why would one person get an answer of $1 \frac{1}{2}$ and another get $\frac{6}{4}$ to the same problem?

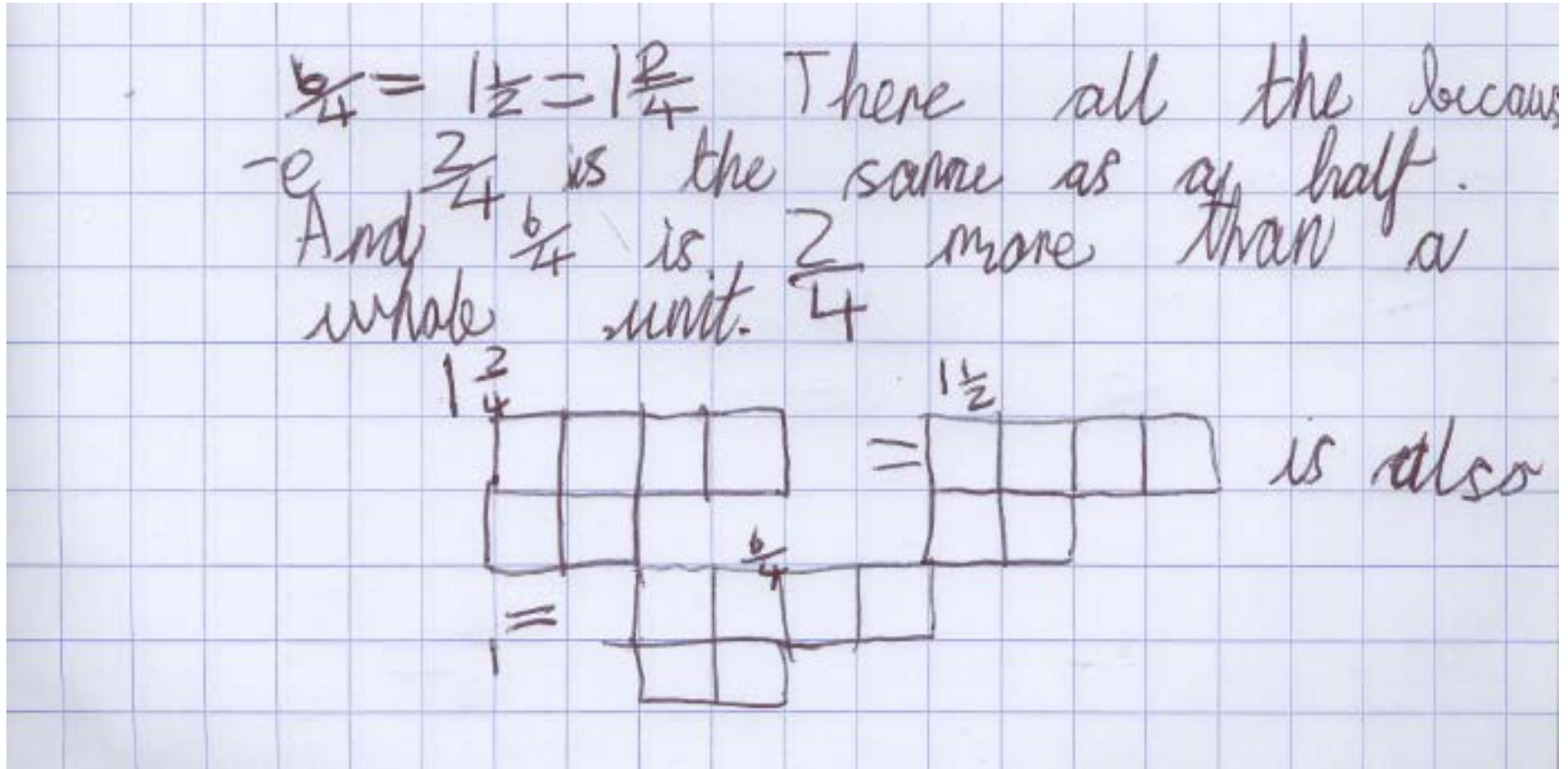
Handwritten explanation on grid paper:

$\frac{6}{4} = 1 \frac{2}{4} = 1 \frac{1}{2}$ because $\frac{2}{4}$ is the same as 1 so $\frac{6}{4} = 1 \frac{1}{2}$ or $\frac{3}{2}$

Why would one person get an answer of $1 \frac{1}{2}$ and another get $\frac{6}{4}$ to the same problem?



Why would one person get an answer of $1 \frac{1}{2}$ and another get $\frac{6}{4}$ to the same problem?



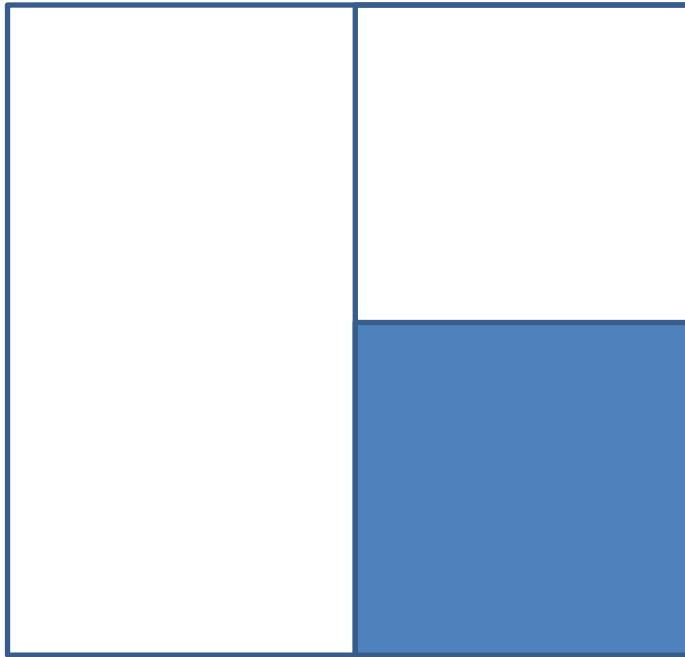
Why would one person get an answer of $1 \frac{1}{2}$ and another get $\frac{6}{4}$ to the same problem?

$\frac{6}{4}$ and $\frac{2}{4}$ are the same
because they is no
difference its just how
you write it

Common misconceptions/errors in maths 1

- $46 - 28 = 22$
- $50\% > \frac{4}{5}$
- $8.35 > 8.5$
- 20 minutes less than an hour is 80 minutes

Common misconceptions/errors in maths 2

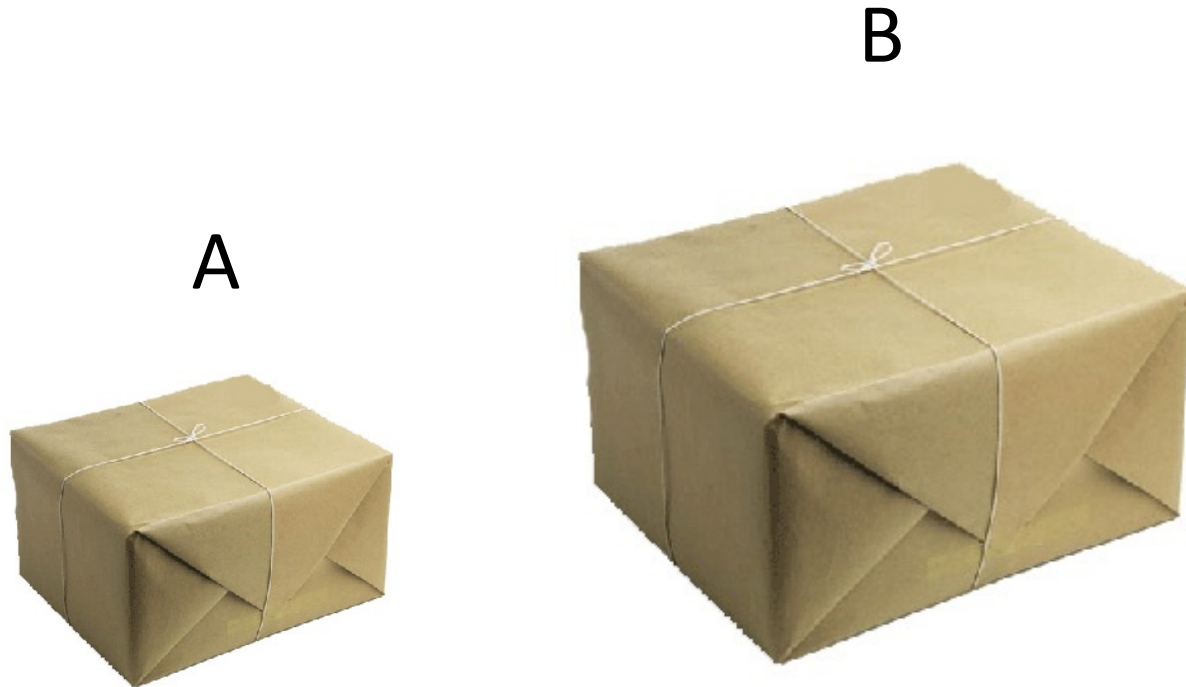


$1/3$ is shaded



9 minutes to 1

Common misconceptions/errors in maths 3



Parcel B is heavier than parcel A

Principles

- Making errors is a natural part of learning mathematics
- Errors provide insights into children's understanding of mathematical ideas
- If errors are not made visible, a teacher cannot address them
- Children don't get "confused" by talking about mistakes

Strategies for Responding to Errors

- Encourage language of agreeing and disagreeing with ideas rather than saying that someone is right or wrong
- Thank children for bringing up something that is potentially confusing for others
- If children don't raise potential errors, refer to another class you taught where some children thought that.... Ask: Why might someone think that
- Follow up by discussing “how can we help the children in this class to remember that...?”

Worked Example in a Textbook from Cyprus

Constantinos's father bought 3 pizzas for his son's birthday party.
The kids ate $1\frac{1}{3}$ pizzas. How many pizzas were left?



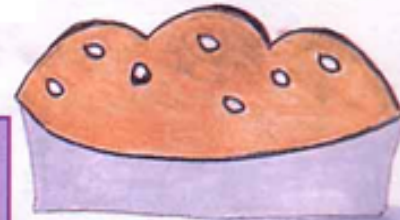
Color the pizza that was eaten.



There were $2\frac{1}{4}$ apple pies in the pan. The kids ate $1\frac{3}{4}$ apple pies.

How many apple pies were left?

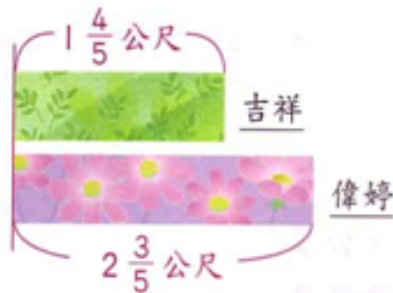
Color the part that kids ate.



Worked Example in a Textbook from Taiwan

2

How many fewer meters of colorful belts did Gi-Wen use compared to Wen-Ting? Write the corresponding mathematical expression and then find the answer.



We cannot subtract $\frac{4}{5}$ from $\frac{3}{5}$.
What should we do?

$$2\frac{3}{5} - 1\frac{4}{5} = (\quad)$$


$2 - 1 = 1$, $\frac{3}{5} - \frac{4}{5}$
不夠減，怎麼辦？

I converted all the mixed fractions into improper fractions.

I converted $2\frac{3}{5}$ to $1\frac{8}{5}$.



$$\begin{aligned} 2\frac{3}{5} - 1\frac{4}{5} \\ = \frac{13}{5} - \frac{9}{5} \\ = \frac{4}{5} \end{aligned}$$



$$\begin{aligned} 2\frac{3}{5} - 1\frac{4}{5} \\ = 1\frac{8}{5} - 1\frac{4}{5} \\ = \frac{4}{5} \end{aligned}$$

Worked Examples in 2 Textbooks from Ireland

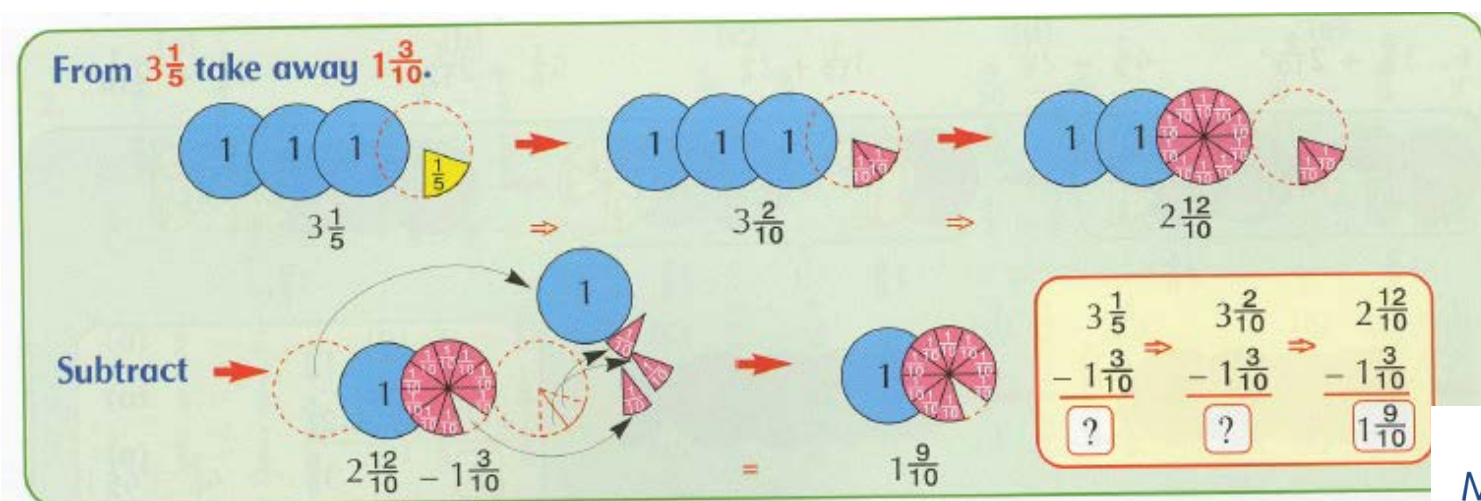
Example 2: $4\frac{1}{6} - 2\frac{3}{4}$

$$\frac{1}{6} = \frac{2}{12}$$

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$$

$$4\frac{1}{6} = 4\frac{2}{12} = 3\frac{14}{12} \quad (\text{by renaming})$$

$$- 2\frac{3}{4} = - 2\frac{9}{12} = - \frac{2\frac{9}{12}}{1\frac{5}{12}}$$



From Charalambous et al, 2010

Algorithms for

- Addition
- Subtraction
- Multiplication
- Division

Problem Solving Strategy 1

RUDE

- Read the problem
- Underline the key words
- Draw a diagram of the problem
- Estimate your answer and then solve the problem

Problem Solving Strategy 2

STAR

- Search the word problem (info)
- Translate the words into an equation or picture (plan)
- Answer the problem (solve)
- Review the solution (check)

Problem Solving Strategy 3

LUV2C

- Look
- Underline
- Visualise
- Choose Numbers
- Calculate

Problem Solving in Maths

- Little evidence that such strategies work
- Some evidence that classifying problems into problem-types can be helpful for children with learning disabilities
- Best way to learn problem solving is to practise solving problems
- Skill in problem solving develops slowly over time
- Many textbooks have too many problems, and many of the problems are of poor quality

A Good Problem

- Should leave the solver feeling “stuck” at first
- The maths is what makes the problem problematic
- Relates to the children’s experience
- Connects different maths topics
- Allows children with different attainment levels to achieve success with it
- May take time, even days, to complete
- Requires children to justify and explain their answers and methods

Sample Problem

- I have 10-cent, 5-cent and 1-cent coins in my money box. If I open the box and take out three coins, how much money could I have? How can you be sure that you have found all the possible amounts?

Other Sample Problems

Donal has 6 jelly tots, Diana has 2 and Enda has 4. They want to share them equally. How will they do it? Draw a picture to help explain your answer.

If you did not know the answer to $12 - 7$, what are some ways you could find the answer?

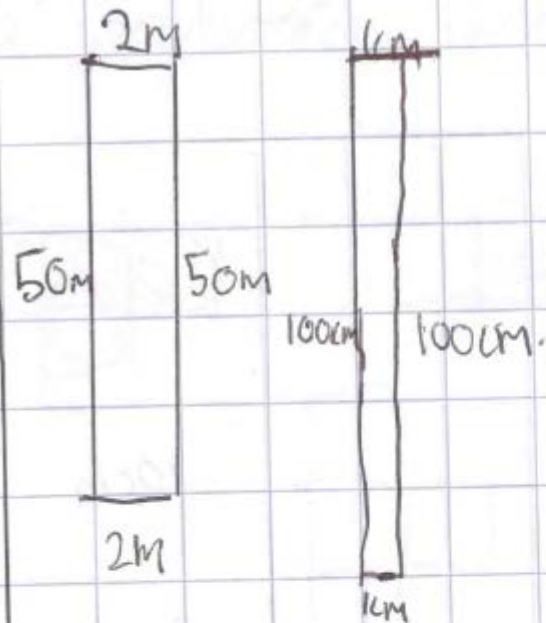
Explain two different ways to multiply 4×276 in your head. Which way is easier to use? Would you use a different way to multiply 5×98 ? Explain why you would use the same or different methods.

Source: Van de Walle

Solve this problem

Dublin Zoo has just received two new sheep for the Family Farm part of the zoo. The zoo keeper wants to build an enclosure for the sheep. She decides that the enclosure must be square or rectangular with an area of 100 square metres.

- Which different configurations could she build?
- How many metres of fencing will she need for each possible design?
- Use your copy or some graph paper to draw all the possible rectangular or square designs.
- Include a key to tell how much each unit on the grid paper equals.
- Which configuration would you recommend that the zoo keeper builds? Why?



We ~~are~~ recommend the 20m X 5m
because there's enough space for
the sheep to move around

And It's not too big and It's not
too small.

I would pick

LG X10

because it has lot

of ~~X~~ room if you

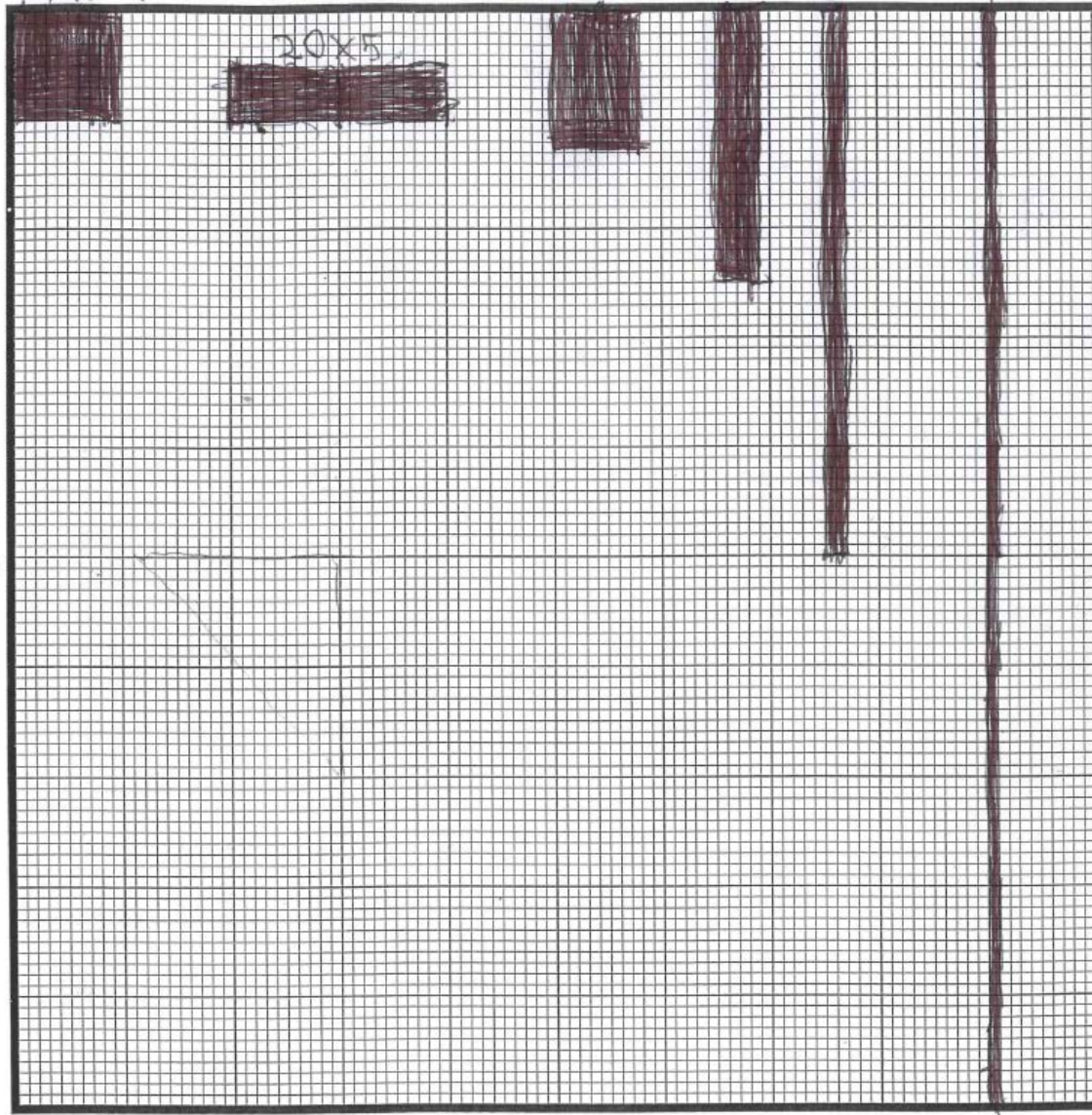
put more sheep in!

I would recommend

loX lo because it is the
most even one.

10m x 10m or 12.5m x 8m. The perimeter of the first one is 40m closely followed by the second on ~~(#)~~ 41m. Because these are so short they are less expensive to fence. They also give lots of width for the animals.

40m
 10x10 10 metres 50m
 41m 58m 104m 202m
 125x8 25x4 50x2 100x1





"CONFiGurations"

7/5/11

① $10m \times 10m = 100m^2 \rightarrow$

② $25m \times 4m = 100m^2 \rightarrow$

③ $20m \times 5m = 100m^2 \rightarrow$

④ $50m \times 2m = 100m^2 \rightarrow$

⑤ $1m \times 100m = 100m^2 \rightarrow$

Sources of Good Problems??

- <http://nrich.maths.org/public/>
- <http://www.nctm.org/>

Here was a site that one of you suggested.

- <http://nzmaths.co.nz/>

Revoicing

- Can you repeat what said?
- Can you put what said in your own words?

Use it to

- Amplify important points
- Check that children are listening
- Clarify what a child is thinking/saying

Aquatics




Swimming Calculations

- What is the length of our local swimming pool?
- If I swam the following number of lengths, how far would I have swum?

Multiplying

1 length
2 lengths
3 lengths
10 lengths
50 lengths
 $\frac{1}{2}$ length

Say how you figured out each one.



The numbers you choose here allow you to differentiate among learners in the class. Some children will do them all and more complicated ones. Others will just do some of these. Try to help each child do better than they think they can do.

Dividing

- If the pool is 25 metres long, how many lengths would I have to swim in order to swim

50 metres
100 metres?
250 metres?
500 metres?
a kilometre?
5 metres?

Say how you figured out each one.

Sort this PE Equipment



Could be sorted by

Criteria

(Categories)

- Colour (yellow, blue, two-colour etc.)
- Type (cone, bat, ball etc.)
- Material (wood, plastic, leather etc.)
- Shape (sphere, non-sphere; round, not round etc.)
- Use (throwing and catching, hitting, marking etc.)
- Property (can roll, can stack, can slide etc.)

Principles when setting a sorting task

- Sorting one set of objects in multiple ways is better than sorting multiple sets of objects in one way each
- Children must choose the criteria and categories for sorting
- It's good if there are some ambiguous categories that require discussion

Figure out the score in points in each of the following GAA matches



Who won each match?
By how much did they win?



This activity introduces children to calculations of the form $(a \times 3) + (b \times 1) = c$ in a concrete way. This is important for understanding place value and multiplication of 2-digit numbers.

Clarifying link to place value and multiplying


Goals	Points	Score in Points
2	7	$(2 \times 3) + (7 \times 1) = 13$

Tens	Units	Value of Number
3	6	$(3 \times 10) + (6 \times 1) = 36$

$$\begin{array}{r} 23 \\ \times 4 \\ \hline 92 \end{array}$$

$$(4 \times 3) + (4 \times 20) = 92$$

GAA score problem

If your team's score in a camogie match was 23 points, what could the score have been in goals and points?  Allows for differentiation

Have you found all the possible solutions?

Some possible solutions to GAA score problem

0 – 23

1 – 20

2 – 17

3 – 14

4 – 11

5 – 8

6 – 5

7 – 2

Could there be other possible scores? Justify your answer.

Could the team have scored exactly 18 points and some goals?
Why? Why not?

Do you notice any pattern in the solutions?

Could be applied to rugby (try, conversion, penalty)

Number Facts/Tables

Take a Minute or Two to Memorise these Addresses and Work Places

Charlie David lives on George Avenue

Charlie George lives on Albert Zoe Avenue

George Ernie lives on Albert Bruno Avenue

Charlie David works on Albert Bruno Avenue

Charlie George works on Bruno Albert Avenue

George Ernie works on Charlie Ernie Avenue

From Dehaene (1997) *The Number Sense* (p. 127)

The Sentences are Roughly Equivalent to:

- $3 + 4 = 7$

Charlie David lives on George Avenue

- $3 + 7 = 10$

Charlie George lives on Albert Zoe Avenue

- $7 + 5 = 12$

George Ernie lives on Albert Bruno Avenue

- $3 \times 4 = 12$

Charlie David works on Albert Bruno Avenue

- $3 \times 7 = 21$

Charlie George works on Bruno Albert Avenue

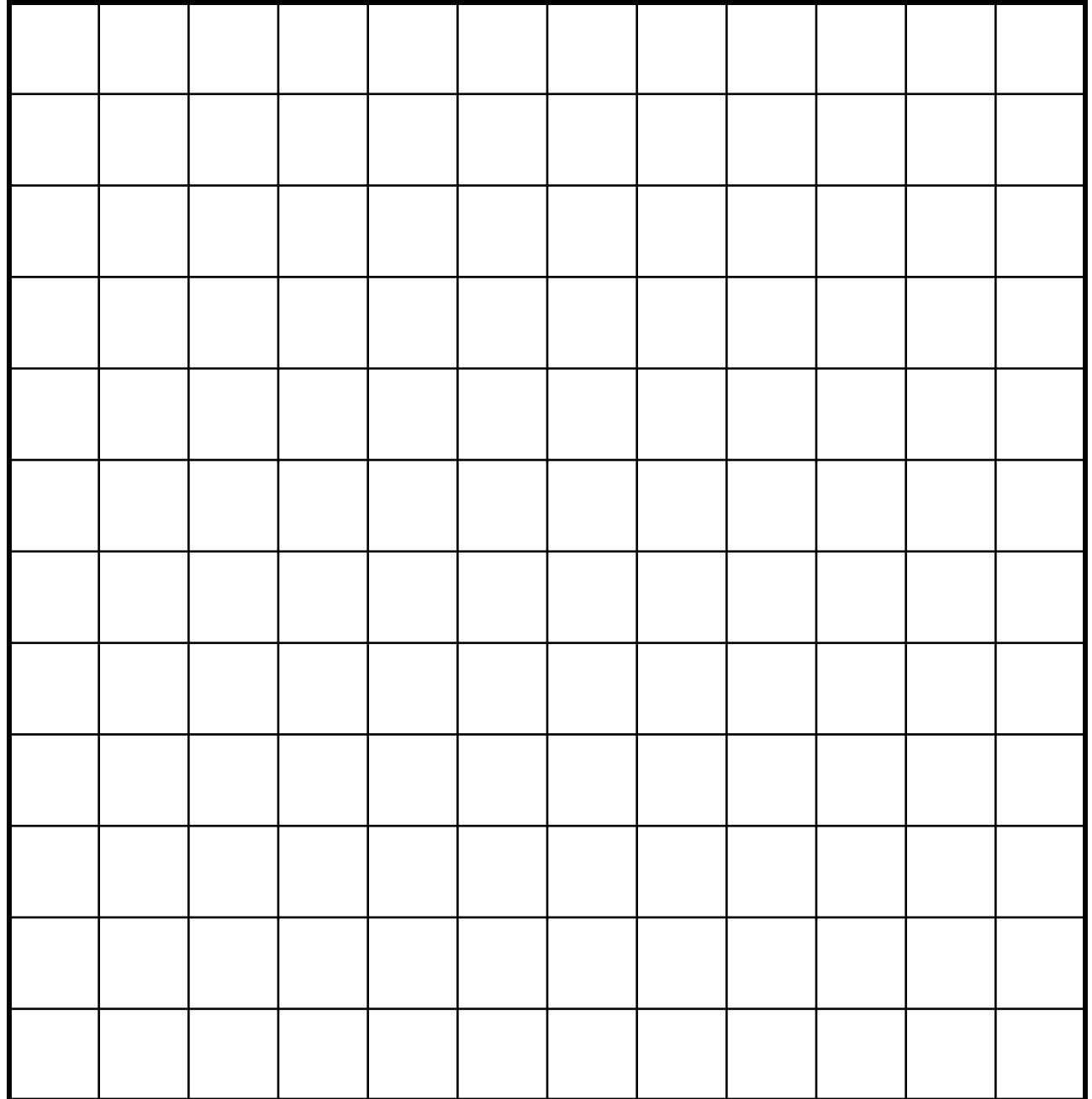
- $7 \times 5 = 35$

George Ernie works on Charlie Ernie Avenue

Addition and Multiplication Number Facts

ADDITION
NUMBER
FACTS IN
FIRST CLASS

Start with a
blank 12x12
grid



Or 6+

+	0	1	2	3	4	5	6	7	8	9	10
0							6				
1							7				
2							8				
3							9				
4							10				
5							11				
6							12				
7							13				
8							14				
9							15				
10							16				

BUILD UP
THE REST
OF THE
ADDITION
SQUARE

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

**MAKE THE
ADDITION
FACTS EASIER
BY
ELIMINATING
THE ...**

**COMMUTATIVE
PAIRS**

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

...THE +0 FACTS

- 0+0
- 1+0
- 2+0
- 3+0
- 4+0
- 5+0
- 6+0
- 7+0
- 8+0
- 9+0
- 10+0

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

...THE +1 FACTS

1+1

2+1

3+1

4+1

5+1

6+1

7+1

8+1

9+1

10+1

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

...THE +2 FACTS

2+2

3+2

4+2

5+2

6+2

7+2

8+2

9+2

10+2

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

...THE TEN AND
FACTS

10+3

10+4

10+5

10+6

10+7

10+8

10+9

10+10

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

**...THE
DOUBLES**

3+3

4+4

5+5

6+6

7+7

8+8

9+9

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

**...THE NEAR
DOUBLES**

3+4

4+5

5+6

6+7

7+8

8+9

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

...THE TEN AND
NINE FACTS
AND FIVE AND
FACTS

6+4

7+3

6+3

5+3

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

...THE
THROUGH TEN
FACTS

7+4

7+5

8+3

8+4

8+5

8+6

9+3

9+4

9+5

9+6

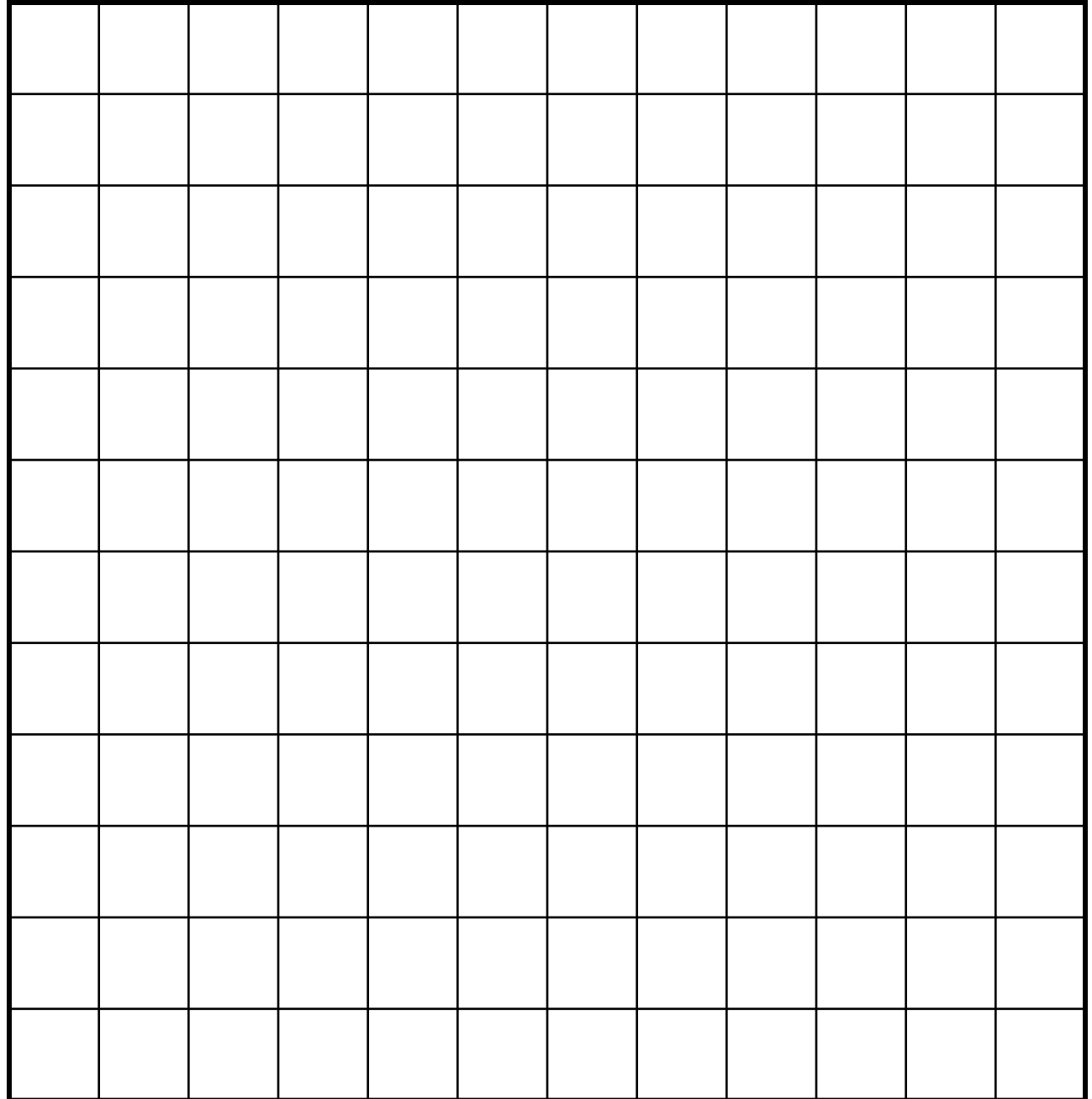
9+7

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

Multiplication

MULTIPLICA
TION
NUMBER
FACTS IN
THIRD
CLASS

Start with a
blank 12x12
grid



SHOW THE
CHILDREN
HOW IT
WORKS BY
GIVING
SOME
EXAMPLES

Eg. 3x

x	0	1	2	3	4	5	6	7	8	9	10
0				0							
1				3							
2				6							
3				9							
4				12							
5				15							
6				18							
7				21							
8				24							
9				27							
10				30							

SHOW THE
CHILDREN
HOW IT
WORKS BY
GIVING
SOME
EXAMPLES

Eg. 6x

x	0	1	2	3	4	5	6	7	8	9	10
0							0				
1							6				
2							12				
3							18				
4							24				
5							30				
6							36				
7							42				
8							48				
9							54				
10							60				

**BUILD UP
THE REST
OF THE
MULTIPLI-
CATION
TABLE**

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**PROMOTE
UNDER-
STANDING
OF MULTI-
PLICATION
FACTS BY
POINTING
OUT:**

**COMMUT-
ATIVE
PAIRS**

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**PROMOTE
UNDER-
STANDING
OF MULTI-
PLICATION
FACTS BY
POINTING
OUT:**

**ZERO
FACTS**

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**PROMOTE
UNDER-
STANDING
OF MULTI-
PLICATION
FACTS BY
POINTING
OUT:**

ONE FACTS

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**PROMOTE
UNDER-
STANDING
OF MULTI-
PLICATION
FACTS BY
POINTING
OUT:**

TEN FACTS

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**PROMOTE
UNDER-
STANDING
OF MULTI-
PLICATION
FACTS BY
POINTING
OUT:**

TWO FACTS

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**PROMOTE
UNDER-
STANDING
OF MULTI-
PLICATION
FACTS BY
POINTING
OUT:**

FIVE FACTS

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**PROMOTE
UNDER-
STANDING
OF MULTI-
PLICATION
FACTS BY
POINTING
OUT:**

**NINE
FACTS**

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**PROMOTE
UNDER-
STANDING
OF MULTI-
PLICATION
FACTS BY
POINTING
OUT:**

**FOUR
FACTS**

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**PROMOTE
UNDER-
STANDING
OF MULTI-
PLICATION
FACTS BY
POINTING
OUT:**

**THREE
FACTS
(2 TIMES,
ADD ONE
MORE)**

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**SIX, SEVEN
AND EIGHT
FACTS
REMAIN.**

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

For slides

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