Validating the Use in Ireland of Adapted U.S. Measures of Mathematical Knowledge for Teaching

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Overview of Presentation

- Mathematical Knowledge for Teaching (MKT) and MKT measures
- Adapting MKT measures for use outside the United States
- Validating the use of the measures in Ireland
- Results of validating the use of the measures
- Challenges of validating the measures
- Discussion

MKT and MKT Measures

Mathematical Knowledge for Teaching

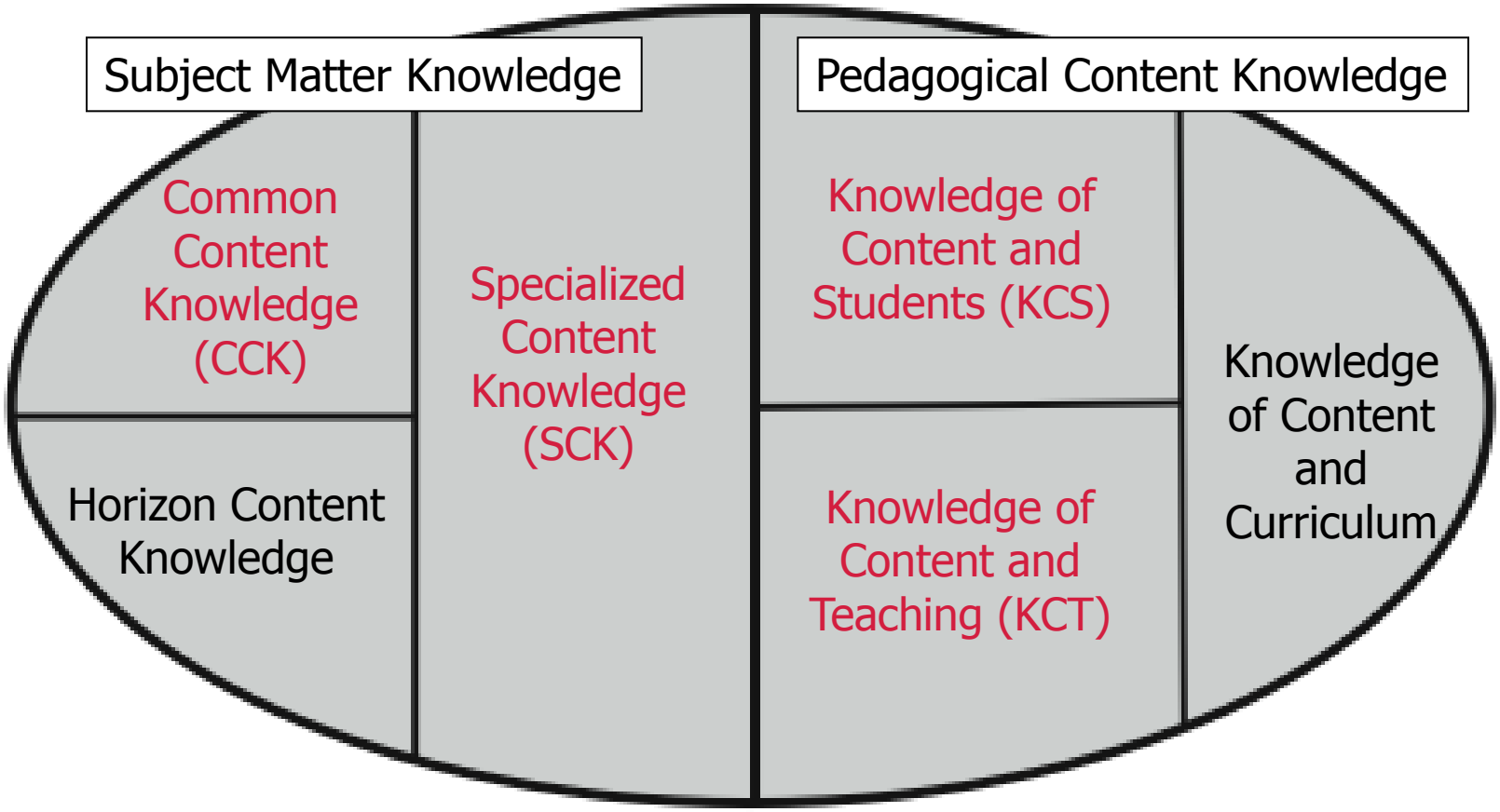
$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline 875 \end{array}$$

How Did this Student Get this Answer?

$$
\begin{array}{r}
35 \\
\times\ 25 \\
\hline
255 \\
+800 \\
\hline
1055
\end{array}
$$

Example from Deborah Ball

Domains of MKT



From Ball, Thames & Phelps (2008)

Sample Item 1

8. Ms. Connell's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

c) Check to see whether 371 is divisible by any prime number less than 20.

d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

Based on item taken from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf.

Sample Item 2

24. Mrs. McKenna is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at some recent classwork to see what they tend to do. She sees the following three student mistakes:

I)
$$\begin{array}{r} 38 \\ 49 \\ +\,6_{1}5 \\ \hline 142 \end{array}$$

II)
$$\begin{array}{r} 45 \\ 37 \\ +\,2_{1}9 \\ \hline 101 \end{array}$$

III)
$$\begin{array}{r} 32 \\ 14 \\ +\,1_{1}9 \\ \hline 64 \end{array}$$

Which have the same kind of error? (Mark ONE answer.)

a) I and II

b) I and III

c) II and III

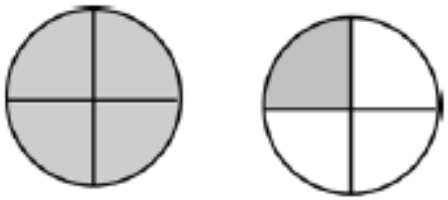
d) I, II, and III

Taken from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf.

Adapting MKT Measures for Use Outside the United States

Need to Adapt Measures 1

5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



a) 5/4

b) 5/3

c) 5/8

d) 1/4

Taken from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf.

Need to Adapt Measures 2

12. You are working individually with Bonny, and you ask her to count out 23 checkers, which she does successfully. You then ask her to show you how many checkers are represented by the 3 in 23, and she counts out 3 checkers. Then you ask her to show you how many checkers are represented by the 2 in 23, and she counts out 2 checkers. What problem is Bonny having here? (Mark ONE answer.)

a) Bonny doesn't know how large 23 is.

b) Bonny thinks that 2 and 20 are the same.

c) Bonny doesn't understand the meaning of the places in the numeral 23.

d) All of the above.

Taken from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf.

Need to Adapt Measures 3

13. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I)
$$\begin{array}{r} {\scriptstyle 1} \\ 38 \\ 49 \\ +\,65 \\ \hline 142 \end{array}$$

II)
$$\begin{array}{r} {\scriptstyle 1} \\ 45 \\ 37 \\ +\,29 \\ \hline 101 \end{array}$$

III)
$$\begin{array}{r} {\scriptstyle 1} \\ 32 \\ 14 \\ +\,19 \\ \hline 64 \end{array}$$

Which have the same kind of error? (Mark ONE answer.)

a) I and II

b) I and III

c) II and III

d) I, II, and III

Taken from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf.

Adapting Measures

- Changes related to the general cultural context
  - Checkers
- Changes related to the school cultural context
  - State assessment
- Changes related to mathematical substance
  - Dollars
- Other changes

See Delaney, Ball, Hill, Schilling & Zopf (2008)

Validating the Use of the Measures in Ireland

Rationale for Validity

- Raise learners' attainment
- Mathematics teaching
- Claims about teachers' mathematical knowledge
- Performance on multiple-choice questions

Kane's Approach to Validity

- Validity in general is a contested issue
- Its implementation is often disconnected from its conceptualisation

Kane:

1. Propose an interpretive argument saying how the results of a test will be interpreted and used
2. Evaluate the plausibility of the interpretive argument

My Interpretive Argument

1. Teachers used their MKT when responding to the multiple choice items
2. The domain of MKT can be distinguished by the types of knowledge deployed by teachers
3. The MKT items capture the kind of knowledge teachers need in order to teach mathematics effectively

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Inferences of the Intepretive Argument

1. Teachers used their MKT when responding to the multiple choice items

A teacher's response to an item is consistent with the teacher's mathematical reasoning about the item

2. The domain of MKT can be distinguished by the types of knowledge deployed by teachers

Items can be distinguished as belonging to one of the conceptualised domains – CCK, SCK, KCS, KCT

3. The MKT items capture the kind of knowledge teachers need in order to teach mathematics effectively

Teachers' scores on the measures are related to the mathematical quality of their instruction

Inferences of the Intepretive Argument

1. Teachers used their MKT when responding to the multiple choice items (Elemental assumption)

A teacher's response to an item is consistent with the teacher's mathematical reasoning about the item

2. The domain of MKT can be distinguished by the types of knowledge deployed by teachers (Structural assumption)

Items can be distinguished as belonging to one of the conceptualised domains – CCK, SCK, KCS, KCT

3. The MKT items capture the kind of knowledge teachers need in order to teach mathematics effectively (Ecological assumption)

Teachers' scores on the measures are related to the mathematical quality of their instruction

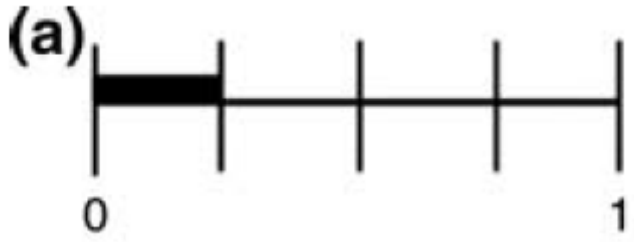
Evaluating the Assumptions

- Convenience sample of 100 Irish teachers responded to pilot test of adapted MKT measures and 5 participated in follow-up interviews
- National sample of 501 teachers completed a test of MKT
- 10 teachers completed a test of MKT and had four maths lessons videotaped

Evaluating the Elemental Assumption 1

- Were teachers' written responses to adapted items consistent with their mathematical reasoning about the items?
- Interviews with five teachers in pilot study about 17 items
- In 74% of responses teachers' reasoning was consistent with their written responses
- In 16.5% of responses it was not possible to determine if teachers' reasoning was consistent or not
- In 9% of responses, teachers' reasoning was not consistent with their written responses

Evaluating the Elemental Assumption 2



How many fractions are there between 0 and 1?

Evaluating the Structural Assumption

- Do the items reflect the conceptual organisation of the MKT theory with regard to the domains of CCK, SCK, KCS and KCT?
- Conducted exploratory and confirmatory factor analyses.
- With a three-factor confirmatory model, three factors could be identified: content knowledge, algebra and some KCS items loaded on a third factor.
- Similar to U.S. Findings.
- BUT the factors are highly correlated among themselves – suggests a higher-order factor
- Perhaps the items don’t measure the domains well or maybe the specification of the domains needs to be modified

Evaluating the Ecological Assumption

- Are the teachers' MKT scores related to the mathematical quality of their instruction?
- "Mathematical quality of instruction" (MQI): "mathematical content available to students during instruction" (Hill et al, 2008; LMT, 2011)
- Global lesson score (Low – medium – high)
- 32 features of mathematical instruction (codes):
  - Teacher's knowledge of the mathematical terrain (e.g. use of technical language, presence of explanations)
  - Teacher's use of mathematics with students (e.g. responding to errors, use of representations)
  - Teacher's use of mathematics to teach equitably (e.g. amount of time spent on maths, explicitness about maths language and practices)

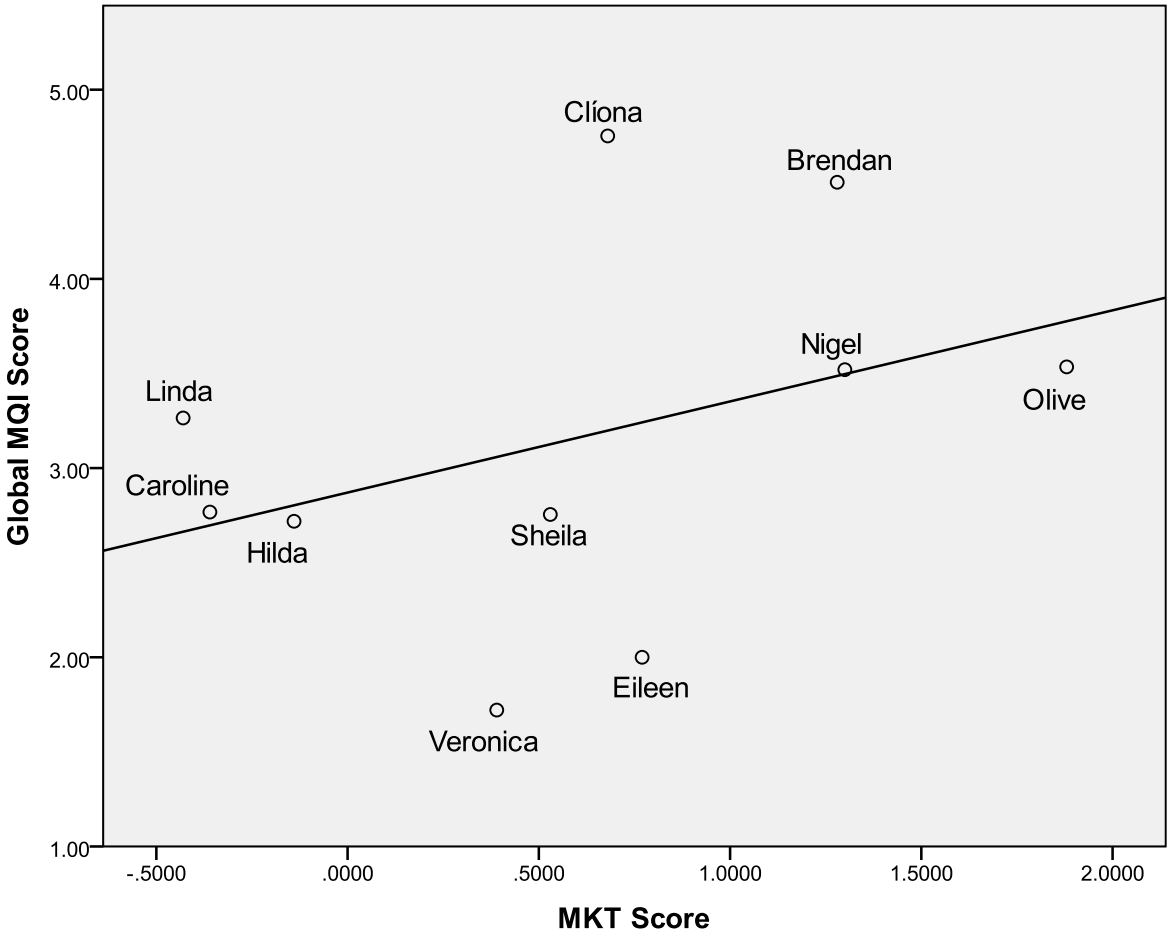
Coding Videotapes for MQI

- Lessons divided into 5-minute clips
- Watch entire lesson
- Watch the lesson again and individually code each 5-minute clip
- Reconcile codes with a partner
- Inter-rater reliability varied from 65% to 100%
- Coding: Make two choices:
  - Feature present or not present?
  - Presence/non-presence appropriate or inappropriate

Try Some Coding

Results of Evaluating the Ecological Assumption

- 10 teachers were videotaped and did the MKT test
- MKT test scores scaled to have a mean of 0 and a standard deviation of 1.
- Score of 0, indicates a 50% likelihood of responding correctly to an item of average difficulty
- Convenience sample
- All teachers between the 36th and 97th percentile of Irish teachers in terms of MKT
- Six in the top quartile of Irish teachers
- Looked for a correlation between MKT and MQI



A regression line fitted to a scatterplot of teachers' scores on MKT and MQI

From Delaney, 2012

Interpretation

- Either the MKT measures are not tapping into the knowledge that teachers use in practice or the MQI instrument is not sensitively measuring the mathematical quality of the instruction observed
- But a correlation was found between MKT and MQI in a study in the United States

Possible Reasons for low MKT/MQI Correlation

- Uneven distribution of teachers on the MKT scale
- MKT measures were from strands of number, algebra and geometry but teachers taught lessons from measures and data strands as well
- Various grade levels taught
- Small sample size
- 30% of lessons were coded by only one coder and margin of error may have been higher than acceptable

Evaluating the Interpretive Argument

- **Elemental Assumption:** Yes, written responses mostly consistent with mathematical reasoning
- **Structural Assumption:** No, similar factor structure to U.S. But distinct domains of CCK, SCK, KCS and KCT not apparent in factor analysis
- **Ecological Assumption:** No, not a strong correlation between adapted measures of MKT and MQI among this sample of Irish teachers

Conclusion

- Conceptualising and measuring teachers' mathematical knowledge is problematic
- Validating the use of adapted measures is challenging

Challenges of Validating Use of Measures

- Conceptualising of MKT – how much has to do with theory and how much to do with the Irish setting?
- Process is costly in terms of time and expertise
- Several variables may affect correlation of MKT and MQI
- Resources not available to recruit a random, national sample of teachers for video study
- How well do the MKT and MQI instruments relate to knowledge needed and used by Irish teachers?

Finally

- TEDS-M study of mathematical knowledge of pre-service teachers in over 20 countries
- Much interest in international studies of students' knowledge (PISA and TIMSS). Work to be done before teachers' knowledge can be compared – and validated - across countries

For More

Delaney, S (2012) A validation study of the use of mathematical knowledge for teaching measures in Ireland. ZDM Mathematics Education.

Special issue of ZDM

Slides: www.seandelaney.com

Discussion

Notice

Studying Mathematical Knowledge for Teaching: A Case of Using U.S. measures in Ireland

- In the United States, the theory of mathematical knowledge for teaching (MKT) has been used as the basis for developing multiple-choice measures of teacher knowledge which can be administered to large groups of teachers. These measures are designed to tap into mathematical knowledge used when teaching. But because they are based on the practice of teaching in the United States, they might be unsuitable for use in other settings.

- This seminar will describe the use of adapted MKT measures to study Irish primary teachers' mathematical knowledge. The measures were administered to a national sample of 501 primary teachers, and a follow-up video study was used to validate the use of the items in Ireland. In presenting his research findings, the presenter will explain the theory of MKT, outline its relationship to the practice of mathematics teaching, and identify matters that arise when using measures based on the theory in non-U.S. settings.