# Annotated Pupils' Book 



Institute of Education

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## Section One: 8 Bckyround information

## Introduction

This is a replacement unit for teaching area in fifth class. The unit may also be helpful for teaching area to pupils in fourth or sixth class and in a mixed class, two or more classes could work on the topic. However, it would be important for a fourth class child to be paired with someone in an older class. The unit was developed based on lessons about area taught to children in fifth class in a mathematics laboratory environment. The children can learn about area by solving the problems in the unit, and develop the mathematical language associated with the topic, through the use of judicious questioning and prompting from the teacher. Such prompting will help the children to develop their thinking by using the mathematical language associated with the topic.

The purpose of this unit is to help you to support the children in your class to think and talk about mathematics, using the topic area. In order to get the most out of working on this unit, the following three principles may be useful to share with your pupils:

1 You can learn a lot from your classmates
2 If you understand something, help others to understand it
3 It's good to ask questions or say "l'm not sure" if you don't understand

## Rationale

The approach that is recommended for the unit differs to the approach that is often followed when teaching maths. First, there are fewer problems to be solved - in most cases just one a day. However, the problems are chosen so that applying concepts and solving the problems can lead to productive class discussions. The idea is that the pupils will learn maths from doing the problems, rather than learning the maths in order to do the problem. The problems are designed so that the children can integrate ideas they have and connect them to situations in their environment.

Second, instead of demonstrating to the pupils how to solve the problem(s) on a day, we suggest that you read over the problem with the pupils and make sure that they understand what the problem is asking them to do. Then let them try their best to solve it by implementing creative strategies and using procedures they know. It might take the pupils a while to get used to working in this way, so you could let them work in pairs for the first few problems. Monitor their progress while they work on the problem, but try not to intervene to reduce the demand of the problem unless a pupil is struggling to solve even part of it.

Third, because we recommend giving the children freedom to decide how they will solve the problem, it is possible that the children will solve it in different ways. Have each one or each pair share their solution with the class and invite others to comment on the different approaches. You might ask questions such as:

- How did $\qquad$ solve it?
- Do you agree with what $\qquad$ did?
(This is to avoid children saying that $\qquad$ is right/wrong)
- Who can put what $\qquad$ did in your own words?

Fourth, although it might be hard, try not to tell the children if their solution is correct. Instead, try to encourage the children themselves to determine if an answer is correct, using their own reasoning. If their reasoning is faulty, this might be questioned by another pupil or by you.

Finally, at the end of the lesson you could ask the children to write in response to one of the following prompts

- I believe that the correct solution to today's problem is $\qquad$ because $\qquad$ _.
- Today in maths I learned $\qquad$ _.
- Tomorrow/next week in maths l'd like to learn $\qquad$ .
- One question I still have is $\qquad$ .

The children may find this difficult the first time you try it. But over time they will get better at communicating and expressing their ideas. The children can also keep a written record of facts, definitions and the area formula as they learn them to help them understand and recall them when needed.

A Line of Development for Area Matched to the Primary School Currricullum Objectives

| Curriculum Objective | Line of Development |
| :---: | :---: |
| Discover that the area of a rectangle is length by breadth | Explain the concept of area |
|  | Discover the formula for area by identifying patterns between the dimensions and areas |
|  | Explain the difference between area and perimeter |
|  | Apply their knowledge of area and perimeter to solve multi-step problems (based on children's experience) |
|  | Design collaboratively a problem related to the topic |
|  | Work backwards collaboratively from a solution to a problem |
| Estimate and measure the area of regular and irregular 2-D shapes | Name, identify and construct benchmarks for square centimetres/metres |
|  | Estimate and measure the area in square units of rectangles of different sizes and orientations |
|  | Apply the formula to find the area of a rectangle |
|  | Calculate the area of shapes where the side lengths have fractions or decimals |
|  | Apply their knowledge of area and perimeter to solve multi-step problems (based on children's experience) |
|  | Design collaboratively a problem related to the topic |
|  | Work backwards collaboratively from a solution to a problem |
| Calculate area using square centimetres and square metres | Estimate and measure the area in square units of rectangles of different sizes and orientations |
|  | Compare the area of different shapes |
|  | Calculate the area in square units of rectangles of different sizes and orientations |
|  | Apply the formula to find the area of a rectangle |
|  | Calculate the area of shapes where the side lengths have fractions or decimals |
|  | Apply their knowledge of area and perimeter to solve multi-step problems (based on children's experience) |
|  | Design collaboratively a problem related to the topic |
|  | Work backwards collaboratively from a solution to a problem |
| Compare visually square metres and square centimetres | Name, identify and construct benchmarks for square centimetres/metres |
| Recognise that length of the perimeter of a rectangular shape does not determine the area of the shape *6th class | Construct 2 or more shapes that have the same area but different perimeters |
| Estimate and measure the perimeter of regular and irregular shapes | Estimate and measure the area in square units of rectangles of different sizes and orientations |
|  | Explain the concept of perimeter |
|  | Estimate and measure the perimeter in centimetres/metres of shapes other than rectangles |
|  | Calculate the perimeter in centimetres/metres of shapes other than rectangles |
|  | Explain the difference between area and perimeter |
|  | Estimate and measure the perimeter in centimetres/metres of rectangles of different sizes and orientations |
|  | Calculate the perimeter in centimetres/metres of rectangles of different sizes and orientations |
|  | Calculate the perimeter of shapes where the side lengths have fractions or decimals |
|  | Apply their knowledge of area and perimeter to solve multi-step problems (based on children's experience) |
|  | Design collaboratively a problem related to the topic |
|  | Work backwards collaboratively from a solution to a problem |

## Possible misconceptions that children may have when learning area and strategies to avoid or surface the misconception

| Possible Misconceptions | Possible Strategies |
| :---: | :---: |
| CONSERVATION OF AREA |  |
| Area can change if the shape is laid out differently | Use a geoboard and ask the children to show shapes with 12 units. Compare and record the shapes children make. Repeat for shapes with other areas (e.g. 24 units, 32 units) |
| UNITS OF AREA |  |
| Square units may not be used to measure the area of non-rectangular shapes | Use a grid placed over various shapes to show how their area can be measured using full squares and partial squares. <br> Note that square units are used to measure irregular shapes such as fields, counties, countries etc. |
| The squares can be seen as discrete rather than continuous. In other words, the children may not understand that you can halve, quarter or otherwise subdivide each square unit | Encourage the children to measure shapes where the side lengths have fractional or decimal fraction amounts in them. |
| Use linear instead of square units for area measurement. For many children a square is not a unit of area. Because children are used to measuring length with centimetres, they initially believe that area too can be measured using units for measuring length. | Use benchmarks to help reinforce the idea of and the difference between units of measurement of length and area. |
| RELATIONSHIP OF AREA AND PERIMETER |  |
| Confuse meanings of area and perimeter | Display definitions of area and perimeter on the classroom wall. |
| Doubling the lengths of the sides of a square doubles its area | Ask the children to find the area of a rectangle that measures 4 cm long and 3 cm wide. Now predict what the area will be if a rectangle is 8 cm long and 6 cm wide. The children might expect the area to be $24 \mathrm{~cm}^{2}$ (twice the area of the first rectangle). Check the prediction by using the formula ( $1 \times$ w) to calculate the area of the second rectangle. <br> Discuss why the prediction of $24 \mathrm{~cm}^{2}$ was incorrect. This could be done by drawing a copy of the two rectangles and comparing them. |
| Difficulties finding the area of a square given the length of one side | Provide children with problems of the form: "One side of a square measures $\qquad$ centimetres/ metres. What is the area of the square?" |
| FORMULA FOR AREA |  |
| Some children may use a strategy of base + height (or length + width) instead of b xh or I xw . | This may be due to children mixing up the ideas of perimeter and area. <br> It may be because the children don't sufficiently attend to the operation (multiplication) that is required to find area when the length and breadth are known. |
| Students sometimes over generalise the formula for rectangles/parallelograms to other shapes | Be explicit with the children that the formula length by breadth (width) refers to rectangles (including squares) and not to shapes more generally. <br> The formula for the area of a parallelogram is length by perpendicular height. If this is used, be explicit that it can only be used for finding the area of a parallelogram. |

## Teacher checklist (see menu of assessment methods)

In order to help children connect new learning to the mathematics they already know, assessment for learning is necessary. The strategies below are suggestions for both assessment for learning and assessment of learning.

| Learning Objective | Possible Assessment Approach <br> See Assessment in the Primary School Curriculum-Guidelines for Schools (NCCA) for further information |
| :---: | :---: |
| THE CHILD CAN: |  |
| 1 Explain the concept of area | Concept mapping |
| 2 Name, identify and construct benchmarks for square centimetres | Each child keeps a list of benchmarks in a mini-portfolio |
| 3 Name, identify and construct benchmarks for square metres | Each child keeps a list of benchmarks in a mini-portfolio |
| 4 Estimate and measure the area in square units of rectangles of different sizes and orientations | Teacher designed tasks and tests |
| 5 Compare the area of different shapes | Portfolio assessment |
| 6 Calculate the area in square units of rectangles of different sizes and orientations | Teacher designed tasks and tests |
| 7 Discover the formula for area by identifying patterns between the dimensions and areas | Self-assessment |
| 8 Apply the formula for area to measure a rectangle that measures $5 \mathrm{~cm} \times 8 \mathrm{~cm}$ | Teacher designed tasks and tests |
| 9 Explain the concept of perimeter | Concept mapping |
| 10 Estimate and measure the perimeter in centimetres/metres of shapes other than rectangles | Conferencing |
| 11 Calculate the perimeter in centimetres/metres of shapes other than rectangles | Teacher designed tasks and tests |
| 12 Construct 2 or more shapes that have the same area but different perimeters | Teacher observation |
| 13 Explain the difference between area and perimeter | Concept mapping |
| 14 Estimate and measure the perimeter in centimetres/metres of rectangles of different sizes and orientations | Teacher designed tasks and tests |
| 15 Calculate the perimeter in centimetres/metres of rectangles of different sizes and orientations | Teacher designed tasks and tests |
| 16 Calculate the area of shapes where the side lengths have fractions or decimals | Teacher designed tasks and tests |
| 17 Calculate the perimeter of shapes where the side lengths have fractions or decimals | Teacher designed tasks and tests |
| 18 Apply their knowledge of area and perimeter to solve multi-step problems (based on children's experience "real life") | Self-assessment (following class discussion of problems) |
| 19 Design collaboratively a problem related to the topic | Portfolio assessment |
| 20 Work backwards collaboratively from a solution to a problem | Self-assessment |

## Definitions

Definitions play an important role in mathematics at all levels. Perhaps the most basic reason for this is that a definition provides a way for everyone taking part in a lesson to know and agree on exactly what mathematical objects are currently under discussion (e.g. square, even number, average... ). That is, when we're discussing a square, we're not discussing just any four-sided figure: it has special properties. This makes it important that all definitions used in maths lessons are clear, understood and used by everyone in the class. It is important to distinguish the mathematical meanings of words from their everyday meanings. Words such as face, area, metre, and many more have mathematical and non-mathematical meanings.

Here are some definitions of maths terms that will be useful when working through the replacement unit. Multiple meanings are given for each term so that you can discuss the different definitions with the children and use the one that is most accurate and most comprehensible for the children. It would be good to display an agreed "working definition" of key terms for reference in the classroom.

## Sources for the definitions have been abbreviated. A full list of sources is provided at the end of this section.

## Conjecture

- A proposition which is consistent with known data, but has neither been verified nor shown to be false. It is synonymous with hypothesis (WH)
- The formation of conclusions from incomplete evidence; guess. (CD)


## Perimeter

- The sum of the length of the sides of a figure or shape (PSC)
- The length of the boundary of a plane region (MT) - from Greek 'peri' meaning 'around' and 'metron' meaning 'measure'
- The distance around the edge of a plane figure (2D shape) (MD)
- The length of the boundary (DH)
- The length around the edge of a polygon [closed 2D figure] or closed curve (MK) \#
- The distance around the outside of a shape (PMH)


## Area

- The amount of surface (MT)
- Amount of material needed to cover it completely (WH)
- The extent of a surface measured in units of length squared (MD)
- Measure of the amount of surface in square units (MK)
- The surface covered by any plane shape (PMH)
- The surface covered by any 2D shape
- The amount of a plane enclosed by a 2D shape measured in square units (NC)


## Millimetre

- A unit of length which is one-thousandth of a metre (MT)


## Centimetre

- A unit of length which is one-hundredth of a metre (MT)


## Square centimetre

- A unit of area equivalent to that contained in a square region with sides of length 1cm (MT)


## Metre

- A base unit of length (MT)


## Square metre

- A unit of area equivalent to that contained in a square region with sides of length 1 m (MT)


## Regular shape

- Having all faces or sides of equal size and shape (MD)
- Any polygon with all sides congruent and all angles congruent (MT)
- A plane (2D) shape which has sides of equal length and equal angles (PHM)


## Irregular shape

- Have unequal sides and/or unequal angles
- A polygon where not all the sides are congruent and not all the angles are congruent (MT)


## Length

- Length is the measure of an object from end to end (PMH)
- The measure of an object in one dimension from end to end (MT)
- In a rectangle the greater of the two dimensions is usually called the length (MD)
- The distance across (MT)


## Width / Breadth

- In a rectangle the smaller of the two dimensions is usually called the breadth (MD)
- The distance across (MT)


## Dimension

- The dimension of an object tells us how many independent directions we can move along from different points on that object'. A surface in 3-D space is a 2-D object, e.g. the surface of a sphere, the surface of the sea, each face of a cube or any other regular solid. In each case, there are two independent directions in which we can move while remaining on the surface. For example on a sphere, we can move along our current line of latitude or along our current line of longitude. Any other motion on the sphere would be a combination of these two: this is the meaning of 'independent directions'.
- A property that can be measured in space where 1-D is linear, 2-D is planar, and 3-D is spatial. (MT)
- A measurement of length, breadth or height relating to a line, two-dimensional shape or three-dimensional shape (PMH)
- The number of coordinates needed to represent the points on a line, shape, or solid. A plane figure is said to be two dimensional; a solid is three-dimensional. (MD)
- The size of a plane figure or solid. The dimensions of a rectangle are its length and width. (MD)
- The dimension of an object is a topological measure of the size of its covering properties. Roughly speaking, it is the number of coordinates needed to specify a point on the object. For example, a rectangle is two-dimensional, while a cube is threedimensional. (WH)


## Sources of definitions. Some have been abbreviated from the originals:

## WW: Elementary and Middle School Mathematics by Van De Walle

MT: The Complete Handbook of Maths terms by Bana, Marshall and Swan
WH: Wolfram MathWorld website, http://mathworld.wolfram.com
DH: Mathematics Explained for Primary Teachers by Derek Haylock
MD: The facts on file dictionary of mathematics by Daintith \& Clark
MK: Mathematical knowledge for primary teachers
PMH: The New Primary Mathematics Handbook by O'Brien and Purcell.
NC: NCCA Glossary (http://action.ncca.ie/media/990/full_glossary.pdf)
CD: The New Collins Concise English Dictionary
PSC: Primary School Curriculum Mathematics

# Section Two: Tasks and Annotations 

## Task

## Line of Development

- Calculate the area in square units of rectangles of different sizes and orientations


## Answer Key

All the possible rectangles that can be made are:
1 unit wide and 36 units long
2 units wide and 18 units long
3 units wide and 12 units long
4 units wide and 9 units long
6 units wide and 6 units long

## Linkage

Number: allow the children to discover that the dimensions of the rectangles are the same as the factors of 36 .


## Lesson Development

After the children work on this task, here are some questions they might be asked:

- What have you noticed about the area of each rectangle?
- If each rectangle were a bar of chocolate, which one would you prefer to eat? Why? Convince the class of your answer.
- Could we record our findings on a table? What information might we want to include on a table to help us record our findings?
- If you had 36 bigger square tiles, will the area of the rectangles that you made earlier be the same as the rectangles you made using the bigger square tiles? Explain your answer. This could be followed by a discussion on the need for a standard unit of measurement.

> Geoboards could also be used instead of square tiles or squares in a copybook.

> Note the use of the word "different" before rectangles. The children may ask if a rectangle that is 2 tiles across and 18 tiles down is different to one that is 18 tiles across and 2 tiles down. These are not considered to be mathematically different.
The term dimension may be new to children but is worth introducing and could be related to their knowledge of 2-D and 3-D shapes. See definitions section. In this case the term is used to refer to length and width/breadth (and not to area).

## Task

## Line of Development

- Explain the concept of area
- Compare the area of different shapes


## Answer Key

Rectangle C has an area of $28 \mathrm{~cm}^{2}$ and takes up the most space.
However, children are not required to come up with the measure, but a reason that justifies their choice.
The other areas are:
Rectangle A: $20 \mathrm{~cm}^{2}$
Rectangle B $24 \mathrm{~cm}^{2}$
Rectangle D: $24 \mathrm{~cm}^{2}$
Rectangle E: $25 \mathrm{~cm}^{2}$


## Lesson Development

The purpose of this activity is to encourage the children to measure area using their choice of materials (or none) and then to have them discuss the various ways of doing it. Possible questions include

- How did you find which shape takes up the most space?
- How can you be sure?
- Who did it a different way?
- What do you think about the way $\qquad$ did it? Justify your answer.
- What are the benefits and drawbacks of each way of comparing the space taken up by the rectangles?
- What do you call the space enclosed by a rectangle?



## Task 3 息

## Line of Development

- Explain the concept of area
- Name, identify and construct benchmarks for square centimetres/metres
- Explain the concept of perimeter


## Answer Key

The areas of the triangles in number 2 are:
Rectangle A: $20 \mathrm{~cm}^{2}$
Rectangle B $24 \mathrm{~cm}^{2}$
Rectangle C: $28 \mathrm{~cm}^{2}$
Rectangle D: $24 \mathrm{~cm}^{2}$
Rectangle E: $25 \mathrm{~cm}^{2}$


## Lesson Development

Here are some questions/tasks that you could ask/set the children in relation to this page:

- Make a list of things that measure about one square centimetre.
- What is the difference between a centimetre and a square centimetre
- Give the children a one-square-centimetre template and 1-square-metre template. Identify 5 things in your classroom or home that would be measured in square centimetres and 5 things that would be measured in square metres. Ask the children to suggest items that they could use as benchmarks for square centimetres (e.g. thumb nail) and square metres (e.g. a square table or a combination of floor tiles)
- List key terms on a chart and display in the classroom. New terms can be added as they are encountered. Definitions can also be modified as necessary. That is why we suggest using the term "working definitions".


#### Abstract

Relating the square unit to the base ten cube is useful in order to give the children a benchmark for it. However, remember to emphasise that it is a face of the cuble and not the cube itself that is a square centimetre.


## It is good to discuss different definitions of terms and to collaborate

 on producing "working definitions" of terms for the class.It can be useful for children to learn the meaning of specific pre-fixes and origins here such as metron (Greek for measure) milli- (Latin term one thousandth) centi- (Latin term for hundredth) and peri- (Greek term meaning around)

Note that some of the terms have dual and closely related meanings. For example, area and length can refer to the thing itself and to a measure of the thing (as in the length of ribbon was 20 cm in length). Furthermore, some of the words can mean something different outside of maths class e.g. the junior infant was playing in the senior children's area.

## A regular shape is one

 where all sides are of equal length and an irregular shape is one where the sides differ in length.> Length, width and breadth can be confusing. If you want to be consistent, you could try the following approach: if the term length is used, it is used in conjunction with width to refer to whichever dimension is longer. Alternatively, the terms width (horizontal) and height (vertical) or width and depth could be used for 2-D shapes or the terms width (horizontal left to right), height (vertical) and depth (horizontal front to back) could be used for 3-D shapes. This may be something you discuss only if the children raise it. Furthermore, it is good to propose and to support a consistent approach to the terms in your class and across the school.

## Task 5

## Line of Development

- Discover the formula for area by identifying patterns between the dimensions and areas
- Calculate the area in square units of rectangles of different sizes and orientations
- Apply the formula to find the area of a rectangle


## Answer Key

## Task 5

Rectangle (a) is $24 \mathrm{~cm}^{2}$, (b) is $40 \mathrm{~cm}^{2}$ and (c) is $21 \mathrm{~cm}^{2}$

## Task 6

This could be done in two ways. The dashes could be extended to the opposite side making an array of centimetres which could be counted or the centimetres in one side length could be multiplied by the centimetres in the adjacent side length.

## Linkage

Multiplication
Money
fractions

## Lesson Development

## Task 5

The children may estimate and measure the rectangles in different ways. For example, some may refer back to the square centimetres mentioned in question 3 , some may use squares in their copies (or on the squared paper at the back of this booklet, and others may multiply the length by the width. The different strategies are to be welcomed and provide material for the children to discuss. It will also mean that the accuracy of the estimates will vary depending on which strategy they choose.

After the children work on the task, here are some prompts for discussion:

- Name objects in your home / classroom whose surface is roughly equal in area to the rectangles you have drawn?
- Ask the children if they notice any pattern between the area of the rectangles and their dimensions? This might help the children to "discover" the formula for finding the area of a rectangle.
- Give the dimensions of other rectangles that would have the same area as rectangles (a) and (b)? Convince the class that the answers given are correct.

The centimetre marks on these shapes are to support the children in figuring out area. Some children may use them to connect the marks and mark out squares centimetres on the shapes. Others may multiply the rectangle side lengths to find the area.

## Possible Extension Activity

- If the cost of a square sheet of gold leaf is $€ 40$, how much would it cost to cover your desk? How much gold leaf would be left over and what would the value of the left over piece be?
- Can you give some examples of desk dimensions where you could use full squares of gold leaf and not have to cut the sheets of gold leaf?


## Task 6

The intention of these diagrams is to help the children grasp the idea of multiplying the length by the width to find the area. However, they may also do this by creating a grid of square centimetres.

It may be useful to introduce the term "formula" to the class. One definition of a formula
is: a mathematical rule used to describe a relationship. E.g. length $x$ breadth $=$ area


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## Task 7

## Line of Development

- Calculate area using square centimetres and square metres


## Answer Key

The answer will vary depending on the area of the desks.


## Lesson Development

If some children are finding it difficult to get started on the problem, these strategies may help:

- Suggest cutting out in paper the dimensions of the gold leaf - using at most one sheet per desk. Estimate how many of these would be needed to cover the desk. Discussion around why an estimate may be useful in this instance but because of the expense of a gold leaf, why an exact answer is required.
- Ask: what are the dimensions of the gold leaf? How can we calculate the area of the gold leaf if we are only given the side length?
- Could we get away with using full sheets of gold leaves to cover the desk exactly and not have to cut at all? Why / why not? How would we know for sure?
- If we know the area of the gold leaf, what other information would we require in order to solve the problem?
- How would we calculate the area of the desk?
- How would we know if our answer was the best?

After the children work on this problem, they could respond to the following questions:

- What different ways did you solve the problem?
- Discussion around why it is necessary to have a standard unit of measurement - $\mathrm{cm}^{2}$ or $\mathrm{m}^{2}$.


## Possible Extension Activity

- If the cost of a square sheet of gold leaf is $€ 40$, how much would it cost to cover your desk? How much gold leaf would be left over and what would the value of the left over piece be?
- Can you give some examples of desk dimensions where you could use full squares of gold leaf and not have to cut the sheets of gold leaf?


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## Task

## Line of Development

- Explain the concept of area
- Compare the area of different shapes
- Calculate the area of shapes where the side lengths have fractions or decimals.


## Answer Key

The area of the top shape is $78 \mathrm{~m}^{2}$ and the area of the lower shape is $76.5 \mathrm{~m}^{2}$.

He might have to think about factors such as storage, availability of a sink etc.


## Lesson Development

This is an opportunity for the children to find the area of non-rectangular shapes and to calculate area using a decimal number.

Again, it is good for the children to work on the problem themselves for a while and to discuss their thoughts about it with their classmates. Note that in the top shape, for example, some children may partition the shape into two rectangles and find the total area by adding the area of each one. Others may build the shape into one rectangle, find its area and subtract the area of the space that was added on.

Children have the opportunity to compare the area of their classroom (or other room in the school if necessary) to the classroom plans in the unit.

As an extension activity for task 8, the children could draw their own classroom to scale on the grid paper.


## Task

## Line of Development

- Explain the concept of area
- Compare the area of different shapes


## Answer Key

The first space measures $84 m^{2}$ and the square space measures $81 \mathrm{~m}^{2}$.


## Lesson Development

The focus in this lesson is on how the children justify their choice of space to their classmates. They can be asked questions such as: how can you be sure that the space is bigger? Of course, they may come up with other reasons for choosing one space over the other. The main point is that they can justify their choice logically and mathematically.

If some pupils say that they would prefer the square because of its regular shape, it should still be evident that they know that they are foregoing some space in favour of the regular shape.

## Thes 9

Then are fwo spaces in the school yard mallable for use ats a vegntable garden for the chidien in tith clest One is a mectang moasuring idm $x$ 6m. The other is a square with a side locith of Mr . The chass want to seinct the bigoir spoce.
6i. Which space mbowed thet thoose?
㐾 Why? Explain your choice to your chnsmaber.
(ofi What are you theanjuing when you wast to com athount of ipace in the two phetory want to compare the


## Task 10

## Line of Development

- Explain the concept of area
- Calculate the area in square units of rectangles of different sizes and orientations
- Construct 2 or more shapes that have the same area but different perimeters
- Calculate the area of shapes where the side lengths have fractions or decimals


## Answer Key

(i) The enclosures with whole number dimensions are:

100 m long and 1 m wide, 50 m long and 2 m wide, 25 m long and 4 m wide, 20 m long and 5 m wide, 10 m long and 10 m wide

If enclosures with fractional or decimal dimensions are included, options such as 12.5 m long and 8 m wide are possible. There is an unlimited number of such possibilities.
(ii) The perimeters need to be calculated for each enclosure. For the whole number options above, they are as follows:

202m, 104m, 58m, 50m, 40m.
(v) Different answers are possible. The cheapest option using whole numbers would be 10 m long and 10 m wide because less fencing is needed. However, you would have a larger viewing area with 100 m long and 1 m wide. The children need to be reminded that the animals will have the same amount of space no matter which option is chosen.


## Lesson Development

This is a problem that the children could solve in pairs.
Before solving the problem you could display a rectangle to the children and ask: if I told you that a rectangular shape had an area of $12 \mathrm{~m}^{2}$, what might the length and the breadth of the rectangle be?

After reading the problem with the whole class, the following questions might be helpful

- What questions do you have about the problem?
- What is the problem asking you to do?
- What do you know?
- What do you need to find out.

It would be good to allow the children to use the enclosed graph paper.
The children can share their findings with the class. It is more important that they can justify their answer mathematically than that they agree on one answer.

> In terms of the "different" enclosures, you could restrict this to enclosures that have whole number measures for length and width. However, a few children may come up with side lengths that contain decimals or fractions.


## Task

## Line of Development

- Explain the concept of area
- Calculate the area in square units of rectangles of different sizes and orientations
- Construct 2 or more shapes that have the same area but different perimeters
- Calculate the area of shapes where the side lengths have fractions or decimals


## Answer Key

The area of the rectangular bar is $27 \mathrm{~cm}^{2}$ and the area of the triangular bar is $28 \mathrm{~cm}^{2}$.


## Lesson Development

The purpose of this problem is to get the children thinking about the relationship between the area of a rectangle and a triangle. One way to prompt the children to think about this would be to cut out a rectangle, find its area and then cut it up in different ways and find the area of parts of the rectangle based on the area of the whole rectangle (e.g. this is half the area of the rectangle because...., this is quarter the area of the rectangle because...). Alternatively, the children might place squared paper over the triangle and calculate how many squares it covers.

It is not intended that the child are introduced to the formula for finding the area of a triangle at this stage.

Ask the children if they could think of other ways of comparing the amount of chocolate in each bar (e.g. by weighing it).


## Task 12

## Line of Development

- Compare the area of different shapes
- Calculate the area in square units of rectangles of different sizes and orientations
- Estimate and measure the perimeter in centimetres/ metres of shapes other than rectangles
- Calculate the perimeter in centimetres/metres of shapes other than rectangles
- Construct 2 or more shapes that have the same area but different perimeters
- Estimate and measure the perimeter in centimetres/metres of rectangles of different sizes and orientations
- Calculate the perimeter in centimetres/metres of rectangles of different sizes and orientations
- What is the difference ...?

Combined
Diameter
Circumference
Scale on maps
Larger scale
Smaller scale
Draw to scale

## Answer Key

(i) Leinster: $19,800 \mathrm{~km}^{2}$, Connacht: $17,788 \mathrm{~km}^{2}$, Munster:24,675km², Ulster:21,552km².
(ii) Louth: $826 \mathrm{~km}^{2}$, Wexford: $2,353 \mathrm{~km}^{2}$, Leitrim: $1,588 \mathrm{~km}^{2}$, Galway: $6,148 \mathrm{~km}^{2}$, Waterford: $1,857 \mathrm{~km}^{2}$,
Cork: $7,499 \mathrm{~km}^{2}$, Armagh: $1,254 \mathrm{~km}^{2}$, Donegal: $4,841 \mathrm{~km}^{2}$
The precise answers here will depend on the sources the children use to get the information. The responses here are based on Wikipedia and there are inconsistencies in the information provided in this source.
(iii) Leinster: $1,527 \mathrm{~km}^{2}$, Connacht: $4,560 \mathrm{~km}^{2}$, Munster: $5,642 \mathrm{~km}^{2}$, Ulster: $3,587 \mathrm{~km}^{2}$
(iv) $24,675 \mathrm{~km}^{2}$
(v) $17,788 \mathrm{~km}^{2}$
(vi) $6,887 \mathrm{~km}^{2}$
(vii) You could add the areas of the 4 provinces.
(viii) Several answers possible (e.g. Louth (826km²),

Carlow ( $896 \mathrm{~km}^{2}$ ) and Dublin ( $922 \mathrm{~km}^{2}$ ): $2,644 \mathrm{~km}^{2}$ ).
(ix) Louth is $826 \mathrm{~km}^{2}$ and Tipperary is $4,305 \mathrm{~km}^{2}$ so it is roughly 5 times the size.


## Lesson Development

When the children come up with their answers, they could be asked:

- Can you say why $X$ county is the largest/smallest county in your province? Please elaborate. Is there another way to say why X is the largest/smallest county in your province?


Uve the intarnat of another coorce ot information in your elassroom to answur the foltowing civestions:
(0) Croosd one provion from ineland. Find the area tit the prosince.
(0) What as the tregeet courtry in your chosen provinoe?
(tir) What is the smalhat vounty in your choten province?
(in) What is the sfllerunce in uma betvenen the largest and the smallest county in your chosen prowince?
(v) Find the ans of the four prowiofen and onder then from mandiest to leygest.
(vi) What is the cifterence in urwa batwean the largest and imalest provice?
(vi) How would you find the total arma of brelund using the informition you hime gathiroif?
(vi) Nume three counties whons combined arsa is winaler than County Cork
(a) About haw mary times bigger in. artia than County Louthial County. Tiponrary?


## Task 13

## Line of Development

- Compare the area of different shapes
- Calculate the area in square units of rectangles of different sizes and orientations
- Estimate and measure the perimeter in centimetres/ metres of shapes other than rectangles
- Calculate the perimeter in centimetres/metres of shapes other than rectangles
- Construct 2 or more shapes that have the same area but different perimeters
- Estimate and measure the perimeter in centimetres/metres of rectangles of different sizes and orientations
- Calculate the perimeter in centimetres/metres of rectangles of different sizes and orientations


## Answer Key

The area of the circular pizza is $28.26 \mathrm{~cm}^{2}$ and the area of the rectangular pizza is $24 \mathrm{~cm}^{2}$ so the circular pizza is better value. The children will most likely not have the precise areas.

## Lesson Development

- Invite children to explain their answer by composing sentences of the form 'I think X mini-pizza is better value because ...' and allow them to justify their answer.
- It is not intended that the children use the formula $\pi r^{2}$ to calculate the area of the circular pizza. They might just count the number of square centimetres that fit in the circle or are covered by the circle.

In your iocal pirta stop you have a choice of a circulter miri pita or a recturigular mirk piata me ahom below. If ther both coat the ano proce tor the sarme toppinge, wisch one is better vatuo? Doscribe thow you know.


## 

## Line of Development

- Discover the formula for area by identifying patterns between the dimensions and areas
- Explain the concept of perimeter
- Construct two or more shapes that have the same area but different perimeters
- Explain the difference between area and perimeter
- Estimate and measure the perimeter in centimetres/metres of rectangles of different sizes and orientations
- Calculate the perimeter in centimetres/metres of rectangles of different sizes and orientations
- Apply their knowledge of area and perimeter to solve multi-step problems
- Design collaboratively a problem related to the topic
- Compare the area of different shapes.
- Calculate the area in square units of rectangles of different sizes and orientations.


## Answer Key

## Task 14

(a) Yes, any one example where the perimeters are the same but the areas are different will suffice. For example, if one field is 10 m long and 10 m wide, and another is 18 m long and 2 m wide. Both perimeters are 40 m but the area of the first one is $100 \mathrm{~m}^{2}$ and the area of the second one is $36 \mathrm{~m}^{2}$.
(b) The answer to question 9 shows that it is possible to have two spaces with the same areas but different perimeters.

## Task 15

5 m .


## Lesson Development

## Task 14

Before the children work on no. 14, you could discuss the following:

- Can you explain the difference between area and perimeter? What is meant by the term 'area'? What is meant by the term 'perimeter'?

After the children have spent some time working on the problem, a class discussion about how they are working on the task may help. Possible questions are:

- How are you working on this problem?
- Why does that help?
- What questions do you have?
- Is anyone thinking differently about the problem?


## Note that this is the first time

 that the notation of $\mathrm{m}^{2}$ occurs in the unit. It would be good to draw the children's attention to this and to ask them how they think square centimetres would be written.
## Possible Extension Activity

- What other dimensions could we have for the length and breadth of this garden so that the area would still be $40 \mathrm{~m}^{2}$ ?
- The children could record their dimensions on a table.
- If we were to put a fence around Mandy's garden and the cost of fencing was €3.50 per metre, what would the cost be of fencing the garden?
- What dimensions for the length and breadth of a garden of $40 \mathrm{~m}^{2}$ would be cheapest to fence?
- Ask the children to collaboratively compose their own problem similar to the problem in their book.


## Task 15

Geoboards or squared paper may be useful for the children in responding to this question. The question gives the children the opportunity to experiment and to develop conjectures about the relationship between area and perimeter.

It is good for children to discuss their responses to the questions and to gain insights into how their classmates approached responding to the questions.





## Task

## Line of Development

- Compare the area of different shapes.
- Calculate the area in square units of rectangles of different sizes and orientations.


## Answer Key

A vertical line could be drawn mid-way along the fifth column. (A diagonal line from the bottom left corner of the lower shape to the top right-hand side of the ninth column is another possibility that one child found.)


## Lesson Development

The purpose of this activity is to encourage the children to think about the units of area as being continuous quantities. It is more important that the children think about a solution and discuss their thinking than that they find a solution.

The terms "row",
"column" and "diagonal" may be useful here



## Extension activity

In Rang 5/6, children meet with scale exercises in maths and SESE. Dot grids are useful for planning the layout of gardens, bedrooms, yards, parks, etc. Ask pupils to design a garage to the following specifications

1 The area must not be greater than 20 square metres.
21 square centimetre on the page represents 1 square metre of the garage.
3 Design a garage that will hold two cars.

## Questions they need the answers to do this:

(i) How long/wide is a car?
(ii) What shape does the garage need to be?
(iii) How much space is needed between the cars?

This task could be adapted so that the children are required to plan a different kind of garage or to plan a bedroom, garden according to specific specifications.




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