Different Ways to Calculate

by Seán Delaney

This article describes some of Constance Kamii's research which shows that teaching children methods for adding, subtracting, multiplying and dividing encourages them to stop thinking for themselves and to 'unlearn' place value. In contrast, if children invent their own ways of calculating, their logical-mathematical knowledge is developed by their making what the primary mathematics curriculum describes as "links between new and existing knowledge." This article appeared in the January/February edition of InTouch, the journal of the Irish National Teachers' Organisation.

How would you calculate the answers to these arithmetic questions?

16+19 52-35 125x3 375÷25

To answer any one of the questions the reader will have employed one of a range of possible methods. Perhaps it was the method learned in school, a modification of a method that was learned or perhaps it was a more ad hoc approach that suited the numbers in the particular question. Most of us are familiar with the methods of subtraction that involve decomposition or 'borrowing and paying back.' These are merely two of several algorithms or recipes for subtraction that if followed properly, will always give the correct answer.

A variety of algorithms exists for each of the number operations (+, -, x and ÷) and the ones that we regularly teach children are those that have been refined and developed over several centuries of use. One woman who believes that primary school pupils should be allowed to use and furthermore be encouraged to invent, their own algorithms is Constance Kamii. Kamii claims that children of first and second class age, who have experience of using numbers, can reinvent ways of calculating without direct instruction. Moreover, her research findings indicate that children who reinvent algorithms or ways of calculating from an early age are more successful and more confident mathematically in later years of primary school.

She explains that if a child is asked to add 35 + 46 the child is generally encouraged to write one number under the other and then to add the units (5+6), to rename the 11 as 1 ten and 1 unit and then to add the tens. However, Kamii argues that if children are *not shown* how to add these numbers and if they have good prior knowledge of number, they are more likely to work from left to right and to say "30+40=70. 5+6 =11. 70+11=81". In other words the children view the numbers in a holistic way and are thus less likely to get 'wild' answers (such as 711). If children are not shown how to calculate they have to think mathematically whereas if they have been shown an algorithm they stop thinking for themselves and tend to rely on memorisation of the rules. When children invent their own algorithms their knowledge of place value is also strengthened.

Last April I met with Constance Kamii and asked her some questions about her research. I voiced the concern that in a typical class some children may not be able to invent an algorithm of their own to calculate answers.

> CK: If they don't invent any that's because they have not reached the level of logical-arithmetical thinking [that is needed] to invent. But, don't worry! Everybody can invent and in a classroom, especially if they are listening to other kids' inventions, many of them think, 'Oh yes, that's a clever way to do it!' and they start imitating.

SD: Does it not save time, resulting in more efficient teaching, if the teacher demonstrates how to do a calculation?

CK: OK. Now, we come to some other research I have done and that is about the harmful effects of teaching carrying and borrowing. And, in the name of education it's really scandalous how we harm children by teaching rules such as carrying and borrowing because of two main reasons. Number one, schools tell you to start with the units [but] children's way is to start with the tens. And so, in order to obey teacher's way of starting with the ones children have to give up their own thinking…The second harm is that children unlearn place value.

SD: Another concern that some people have is that it might be okay for average children in the class or for children who are doing well but how do children who struggle with mathematics respond to being asked to invent their own algorithms?

CK: Well, logical-mathematical knowledge is such that when they can invent, they can invent! It's the same thing for all children. And, in fact, the children who are harmed the most by carrying and borrowing are the low-level, low-ability children. They are harmed more than really bright kids who understand that the 1 in 12 is a ten. In second grade 20% of the children, are not harmed [by being taught a method for adding etc.] because they understand that that's a ten and not a unit. But the lower a child's [mathematical ability] level is the more they are harmed by the rules.

In order to be competent at mathematics children need to develop their logical-mathematical knowledge. This knowledge is constructed by each child from within, in interaction with the environment or as the primary mathematics curriculum puts it "experimentation, together with discussion.. may lead to general agreement or to the re-evaluation of ideas and mathematical relationships". A teacher can promote children's development of such knowledge by posing questions such as, say 17+18 (initially) and

- giving pupils time to think of an answer
- encouraging pupils to share and explain their answers with the class
- allowing pupils to express agreement or disagreement with the answers
- allowing the pupils themselves to decide on the correctness (or otherwise) of the answers.

Kamii argues that this needs to be done in first or second class because "by third [class] the teacher has to make heroic efforts and I won't say it's hopeless but it takes a long, long time to get children to do their own honest thinking, because they just hang on to the easy rule even if it's not understandable, they hang on to that old rule."

If you would like to discuss these issues or other mathematical topics, the *Primary Teachers Mathematics Association* will play host to Constance Kamii from 9:30 to 4:30 on March 22rd 2003 in Mary Immaculate College in Limerick. All teachers are welcome. For information phone 01 805 7722.

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Photograph of Constance Kamii taken from: http://www.nctm.org/dialogues/2000-10/needmore.htm