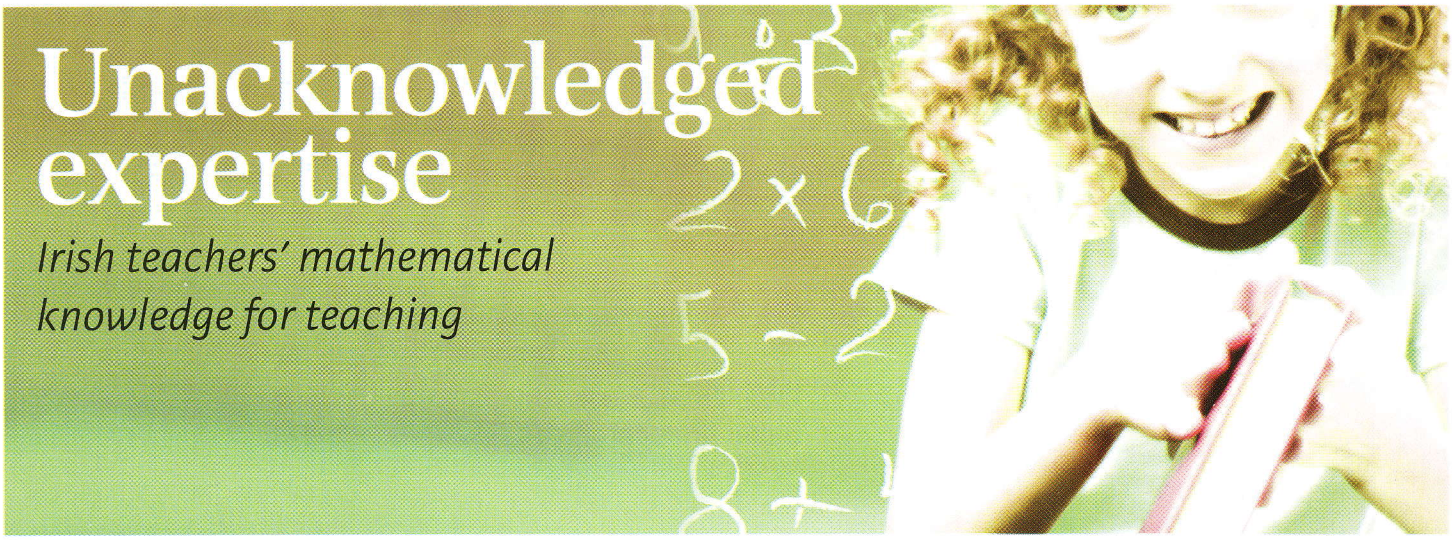
Unacknowledged expertise

Irish teachers' mathematical knowledge for teaching



If you said you were an engineer or a computer programmer most people would assume you knew a lot of maths. But when you say you're a primary teacher, few people automatically consider you to be a maths whiz. Even among themselves, teachers rarely acknowledge or discuss the mathematical knowledge that they use. Such silence seems strange. Think of the mathematical tasks that teachers regularly engage in when teaching: selecting good examples, analysing pupils' errors, asking mathematical questions, answering questions, following different approaches for solving problems, evaluating textbooks and so on. All these activities require mathematical knowledge.

Perhaps the reason we underestimate teachers' mathematical expertise is because little is known about it. The knowledge teachers use differs from the kind of maths taught in secondary school or in university maths courses. It even differs from the maths that primary pupils learn. Instead, teachers use mathematical knowledge that is specialised to the work of teaching.

Recent studies show that primary teaching is mathematically demanding work, and that teachers possess mathematical knowledge not held by engineers, or even by professional mathematicians (eg Ball and Bass, 2003). Ball and her colleagues in the United States developed multiple-choice measures of "mathematical knowledge for teaching" to study mathematics used by teachers. I adapted some of the measures, which are set in teaching contexts (see Figures 1 and 2)[1], and used them to survey over 500 Irish teachers.[2] The purpose of the survey was not to study the knowledge held by any individual teacher, but the mathematical knowledge held by Irish teachers in general.

Variation in teachers' knowledge

Many teachers answered most of the questions correctly, indicating that many Irish teachers possess high levels of mathematical knowledge for teaching. But teachers' scores varied widely. For example, the highest scoring teachers responded correctly to about three times as many items as the lowest scoring teachers did. In other words, levels of mathematical knowledge vary substantially among teachers. Such variation is understandable for many reasons:

- For several years prior to the late 1980s, researchers internationally paid little attention to the topic of teachers' subject matter knowledge.[3]
- Because little was known about the mathematical knowledge needed for teaching, teacher educators at preservice and inservice levels had no research base to inform the design and delivery of mathematics courses for teachers.
- Ireland's teachers are recruited from several sources – long-established colleges of education, an online provider of teacher education, several colleges in the United Kingdom and elsewhere – all of which have different policies about and procedures for developing teachers' mathematical content knowledge.
- It is difficult for teachers to have expert knowledge in all subject areas and the survey only considered knowledge of mathematics.

As well as the variation in mathematical knowledge among teachers, I identified particular areas of strength and difficulty for teachers.

Strengths in Irish teachers' knowledge

Identifying and classifying pupils' mistakes

Teachers generally did well at identifying and classifying pupils' mistakes. Teachers use this knowledge when correcting pupils' work – finding out who is making similar mistakes so that they can respond appropriately. Figure 1 (right) shows an item from the survey that drew on this type of knowledge.

Matching fraction calculations with representations

A second area of strength for Irish teachers was matching fraction calculations with pictorial representations such as area models (eg, a rectangle partitioned into quarters), set models (eg, one quarter of twelve pencils), line segments (eg, a line segment partitioned in quarters) and number lines (eg, the numbers 0, 1/4, 1/2, 3/4 and 1 marked on a line).

Algebra

Algebra questions posed few difficulties. Most teachers responded correctly to tasks such as evaluating the accuracy of various perimeter formulae expressed in terms of w (width) and l (length), and determining how the area of a rectangle is affected when its width is doubled and its length halved.

Mrs Jackson is planning mini-lessons for students around particular difficulties that they are having with subtracting from large whole numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at some recent work to see what they tend to do. She sees the following three student mistakes:

I	II	III
$\overset{4\ \ 12}{\cancel{5}0\cancel{2}}$	$\overset{4\ \ \ \ \ \ 15}{3\cancel{5}00\cancel{5}}$	$\overset{6\ 9\ 8\ 15}{\cancel{7}\cancel{0}\cancel{0}\cancel{5}}$
– 6	– 6	– 7
406	34009	6988

Which have the same kind of error? (Mark ONE answer.)

a) I and II
b) I and III
c) II and III
d) I, II, and III

Figure 1: US item adapted to study Irish teachers' mathematical knowledge for teaching

Areas of difficulty for Irish teachers

Attending to explanations and evaluating understanding

Some topics were more difficult. One difficulty for Irish teachers was following pupils' explanations and checking if their explanations demonstrated understanding of an idea. Figure 2 (right) shows a typical item that taps into this type of mathematical knowledge. It states that anywhere a plus sign – three squares wide and three squares tall – is shaded on a 100-square, the sum of numbers on the row equals the sum of numbers on the column. Four pupils' explanations for why this is true are presented and teachers are asked which explanations show sufficient understanding of why the pattern is true for all similar plus signs.

Let's consider the explanations one at a time – you might want to try responding to the item before continuing:

a) The first explanation states that in any plus-sign shape on the 100-square, the average of the three vertical numbers is the same as the average of the three horizontal numbers. If the averages of two equal-size sets of numbers are equal, then it follows that the sums of both sets of numbers are equal. This response shows evidence of understanding why the pattern is true.

b) The second response simply makes a statement about the specific plus sign shaded on the 100-square. Nothing said explains why this might be true in other parts of the 100-square and the statement does not move much beyond the original pupil's claim.

c) The third explanation uses another relationship between the row and the column to explain why the pattern is true. The pupil notes that the three numbers on both row and column add up to three times the number in the middle. This observation, which is generalised to "no matter where the plus sign is", shows understanding of why the pattern always holds: if the three numbers add up to three times the middle number and the middle number is the same for the row and the column, the sums of the row and the column will be equal.

d) The fourth statement is true but it refers only to the numbers in the column, not to the numbers in the row. In order to show understanding, an explanation must identify a relationship that exists between the column and the row.

Ms Walker's class was working on finding patterns on the 100 square. A student, Lorraine, noticed an interesting pattern. She said that if you draw a plus sign like the one shown below, the sum of the numbers in the vertical line of the plus sign equals the sum of the numbers in the horizontal line of the plus sign (ie, 22 + 32 + 42 = 31 + 32 + 33). Which of the following student explanations shows sufficient understanding of why this is true for all similar plus signs? (Mark YES, NO or I'M NOT SURE for each one.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	**22**	23	24	25	26	27	28	29	30
31	**32**	**33**	34	35	36	37	38	39	40
41	**42**	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

	Yes	No	I'm not sure
a) The average of the three vertical numbers equals the average of the three horizontal numbers.	1	2	3
b) Both pieces of the plus sign add up to 96.	1	2	3
c) No matter where the plus sign is, both pieces of the plus sign add up to three times the middle number.	1	2	3
d) The vertical numbers are 10 less and 10 more than the middle number.	1	2	3

Figure 2: US item adapted to study Irish teachers' mathematical knowledge for teaching

Irish teachers found the 100-square item difficult, especially parts (b) and (d) where they frequently considered incomplete explanations to show adequate understanding of the mathematical idea.

Identifying and applying properties of numbers and operations

Teachers often share rules with students that are only partly true when examined mathematically: "You can't take a larger number from a smaller number" is one example. Another example is that pupils sometimes believe that adding two numbers always results in a bigger number. This 'rule' is clearly true for natural – or counting – numbers (ie 1, 2, 3, 4, 5...) because the smallest counting number is 1 and if 1 is added to itself the sum, 2, is a bigger number. But if the rule is applied to whole numbers (ie, 0, 1, 2, 3, 4, 5...), it's not true because, 0 + 5 = 5, which is not bigger than 5. If the rule is applied to integers (ie ...-3, -2, -1, 0, 1, 2, 3...) it remains untrue because -3+ (-4)= -7 and -7 is less than both -3 and -4. Therefore, the rule is limited. But many teachers believed such 'rules' to be always true, possibly because they considered them only in relation to counting numbers – the first set of numbers pupils meet in primary school.

Matching word problems with fraction calculations

Teachers often help pupils understand operations with fractions, such as 1/2 - 1/3, by putting them in real-world contexts. The tricky part of creating these contexts is being consistent about what the whole unit is. The word problem below mentions both numbers and includes the word "gave" which implies subtraction; but the problem is not solved by calculating 1/2 – 1/3.

Mary had 1/2 a box of sweets and she gave 1/3 of the sweets to her brother. What fraction of a box of sweets did Mary have left?

The problem is that for 1/2, the whole refers to the box of sweets and for 1/3, the 'whole' is the half box of sweets. That word problem would be solved by calculating 1/2 - (1/3 *of a* 1/2) or 1/2 – 1/6. A suitable word problem for 1/2 – 1/3 is one where the whole unit is kept constant, such as:

Mary had a 1/2 crate of apples and she gave 1/3 of a crate of apples to her brother. What fraction of a crate of apples does Mary now have?

Mathematical knowledge and classroom instruction

The tasks listed above are ones which Irish teachers generally found easy or difficult on the survey of mathematical knowledge for teaching. But you might wonder if performance on multiple-choice items is related to how teachers teach. The phrase "mathematical quality of instruction" (Hill et al., in press) refers to classroom practices such as how teachers represent mathematical ideas and connect representations to each other; how they describe, explain and justify mathematical ideas and encourage their pupils to do the same; how accurately teachers use language and how explicit they are in talking about mathematical skills. In short, it refers to "several dimensions that characterise the rigor and richness of the mathematics of the lesson" (p 4).

Ten Irish teachers who completed the survey were videotaped teaching four maths lessons each and the mathematical quality of instruction exhibited in the lessons was rated. In general, teachers with higher scores on the multiple-choice items exhibited instruction of a higher mathematical quality. No doubt, factors other than a teacher's knowledge affect instruction, but teacher knowledge certainly helps. In the United States teacher knowledge has been linked to students making greater gains on standardised test scores (Ball, Hill, & Bass, 2005) but that relationship has not yet been studied in Ireland.

Learning more about teachers' specialised knowledge

I want to finish up by thanking the teachers all around the country who participated in the study described here. Several teachers said they were taking part because they hoped the findings might help their pupils. The study shows that teachers possess and use knowledge that is different to knowledge held by those in other jobs (Ball et al., 2005) and that this professional knowledge makes a difference in instruction. Much more remains to be learned, in Ireland and elsewhere, about teachers' specialised knowledge of mathematics. So, I have to ask: have you ever thought about the amount of mathematical knowledge you use when teaching and about how that knowledge is specific to the work teachers do?

Perhaps examples of mathematical knowledge used when teaching could be shared through the medium of online forums or through the pages of *InTouch*. By recognising and sharing examples of this knowledge, professional development opportunities in maths could be made more relevant for teachers. As well as that, learning more about teachers' mathematical knowledge could help teacher educators equip future teachers with the mathematical knowledge they need to help pupils practise skills such as applying and problem solving, communicating and expressing, and reasoning, as mentioned in the curriculum.

Most of us have been told that concrete materials help pupils learn maths. But a more important resource could be the teacher's own knowledge. Learning more about mathematical knowledge specific to teaching can help to strengthen the professional knowledge base of teaching.

Next month: Read about examples of teachers using mathematical knowledge when teaching

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Notes

1. More items (not adapted for Ireland) available from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf
2. Between June and December 2006, 501 teachers responded to 84 items on the survey form; surveys were completed in the presence of a researcher. 75% of teachers who were asked to participate did so. Every teacher in a random, representative sample of 87 primary schools, chosen from all primary schools (excluding special schools because many of them include students of primary and post-primary school age) was invited to participate.
3. One turning point that is frequently identified was a paper by Shulman (1986) where he described teacher knowledge as the "missing paradigm" in educational research.

INTO/GAA Mini Sevens cont/d from page 27

HOST COUNTIES FOR 2009 GAA/INTO MINI SEVENS

Region	Hurling Camogie	Football
1	Louth	Armagh
2	Antrim	Fermanagh
3	Mayo	Roscommon
4	Waterford	Tipperary
5	Carlow	Wicklow
6	Meath	Longford
7	Dublin	Dublin

KEY DATES FOR 2009 SERIES

24 April All counties will have completed their own competitions.
1 May Forms to be completed by county winners and copies sent to the secretary of the host county and the National Coordinator.
22 May All regional festivals to be completed.
12 June The children who have been selected to play in Croke Park and their teachers will be notified by the National Co-ordinator.


David Naughton in action

PARTICIPATING COUNTIES IN 2009: REGIONS

Region 1	Region 2	Region 3	Region 4	Region 5	Region 6	Region 7
Armagh	Tyrone	Roscommon	Tipperary	Wexford	Longford	Dublin
Down	Derry	Galway	Cork	Wicklow	Westmeath	
Cavan	Donegal	Mayo	Kerry	Laois	Kildare	
Monaghan	Fermanagh	Sligo	Limerick	Kilkenny	Offaly	
Louth	Antrim	Leitrim	Waterford	Carlow	Meath	
		Clare				

To keep up to date with the 2009 Mini sevens series and to see photographs from past series of matches in Croke Park log onto www.scoilsport.org