Mathematics Teaching Practices and Practice-Based Pedagogies: A Critical Review of the Literature Since 2000

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Abstract

Two exciting lines of research have emerged or have been expanded over the past two decades in relation to the content and practice of mathematics teacher education. The first concerns the attempt to theoretically identify and codify practices of teaching in general, and core or high leverage practices, in particular. The second concerns the identification of practice-based pedagogies of teacher education that may encourage the teaching and learning of the latter practices. Reviewing key findings emanating from research that has been undertaken since 2000 in these two realms of mathematics education, we discuss the advances made in rendering practice a key aspect of understanding and improving teaching and teacher education. At the same time, adopting a critical stance, we point to challenges in conducting research in these areas, including the lack of shared language, the absence of an agreed-upon suite of methodologies to empirically examine theoretical arguments advanced in these areas, and the need for stronger and more systematic empirical validation of the potential of teaching practices and practice-based approaches to teacher education. In doing so, we outline open issues worthy of investigation in the next decade in each line of research separately and jointly.

Introduction

In his classic book, *Schoolteacher*, American sociologist Dan Lortie (1975) described teacher isolation as one of the main impediments to teachers' learning in and from practice. Teachers, Lortie lamented, spend much time isolated from other adults, largely interacting only with students. As a result, schools become sites for student learning and only infrequently for teacher learning. For decades scholars have advocated incorporating teacher learning into the work of teaching. Ball and Cohen (1999) called for a practice-based curriculum in teacher education, a curriculum that sets critical examination of the practice of teaching at its core. *How* teachers learn joined the concern about *what* teachers need to learn with an argument that teaching "must be learned in and from practice rather than in preparing to practice" (p. 12). For such learning to take place, "professional development needs to be grounded in the actual tasks, questions, and problems of practice" (p. 20).

Viewing practice as a potential cornerstone of teacher learning, has led to concerted efforts to better understand the work of teaching and the practice-based environments that can support teacher learning for over two decades now. Key to these efforts have been attempts to not simply document the complexities of teaching but to codify and explicitly communicate what is entailed in the work of teaching (e.g., Cohen, 2011; Lampert, 2001). Lampert (2001) provides a comprehensive account of what teaching entails and how its complexity can be managed. Along similar lines, Cohen (2011) identifies questions to consider if teacher learning and consequently instructional

quality is to be improved: "What sort of an endeavor is teaching? What kinds of problems must teachers solve, and how do they solve them? And what would it take to solve them in ways that promote ambitious teaching and learning?" (p. 3).

Two decades after Ball and Cohen's call for a practice-based curriculum in initial teacher training and ongoing professional development, analyzing practice has become a key aspect of understanding and improving teaching and teacher education. Therefore, it seems opportune to review what has been accomplished so far in this area of mathematics education and to consider challenges and open issues for future research. To do this we reviewed research published since 2000 on two cutting-edge lines of research in mathematics teacher education. The first pertains to codifying and understanding teaching practices that are learnable by teachers and teachable by teacher educators, in the service of student learning; the second relates to identifying practice-based pedagogies that facilitate the teaching and learning of such practices.

In what follows, we outline the methods pursued in identifying relevant literature. In the next two sections, we focus on each strand – teaching practices and practice-based pedagogies – discussing what has been accomplished and considering challenges and open issues. We conclude by identifying issues and challenges for future research.

Methods

Figure 1 summarizes the steps taken to identify, annotate and synthesize relevant literature. First, relevant databases and a set of keywords[1] were identified for initially screening suitable sources, guided by clear inclusion and exclusion criteria. Peer-reviewed journal articles, books and chapters, including only texts written in English were sought. Conference proceedings and dissertations were excluded. Chapters from key handbooks in the field[2] were examined to complement the search. We skimmed chapters of these handbooks, looking for themes related to teaching practices and practice-based pedagogies. Attempting to be more inclusive, we included chapters that referred to teaching practice(s), as well as chapters that pertained to teachers' learning in general. Finally, three books related to the themes were included: Lampert's (2001) *Teaching Problems and the Problems of Teaching*, Cohen's (2011) *Teaching and its Predicaments*, and Grossman's (Ed., 2018) book on *Teaching Core Practices in Teacher Education*.

[1] For the first strand, we included the keywords "practices," "teaching practices," "practices of teaching," "core (teaching) practices," "high leverage (teaching) practices," "instructional practices" along with "mathematics"; for the second strand, we considered keywords such as, "approx[imat]* of practice," "represent* of practice," "decomp[os]* of practice," "signature pedagog*," "practice-based pedagog*," "rehearsals", along with "mathematics". We did not include keywords such as "lesson study" or "video-viewing"/"video- clubs" because, as we argue later, we consider them as being included under the overarching umbrella of representations, decomposition and approximations of practice. Additionally, including these terms would result in an even longer list of studies which would be hard to handle in this chapter. We, however, point the reader to (review) studies focusing on these topics.

[2] These included the fourth edition of the *Handbook of Research on Teaching* (2001), the *Handbook of Research on the Psychology of Mathematics Education: Past, Present, and Future* (2006), the *Second Handbook of Research on Mathematics Teaching and Learning* (2007), the four volumes of *The International Handbook of Mathematics Teacher Education* (2008), the third edition of the *Handbook of Research on Teacher Education: Enduring Questions in Changing Contexts* (2008), the fifth edition of the *Handbook of Research on Teaching* (2016), and the third edition of the *Handbook of International Research in Mathematics Education* (2016).

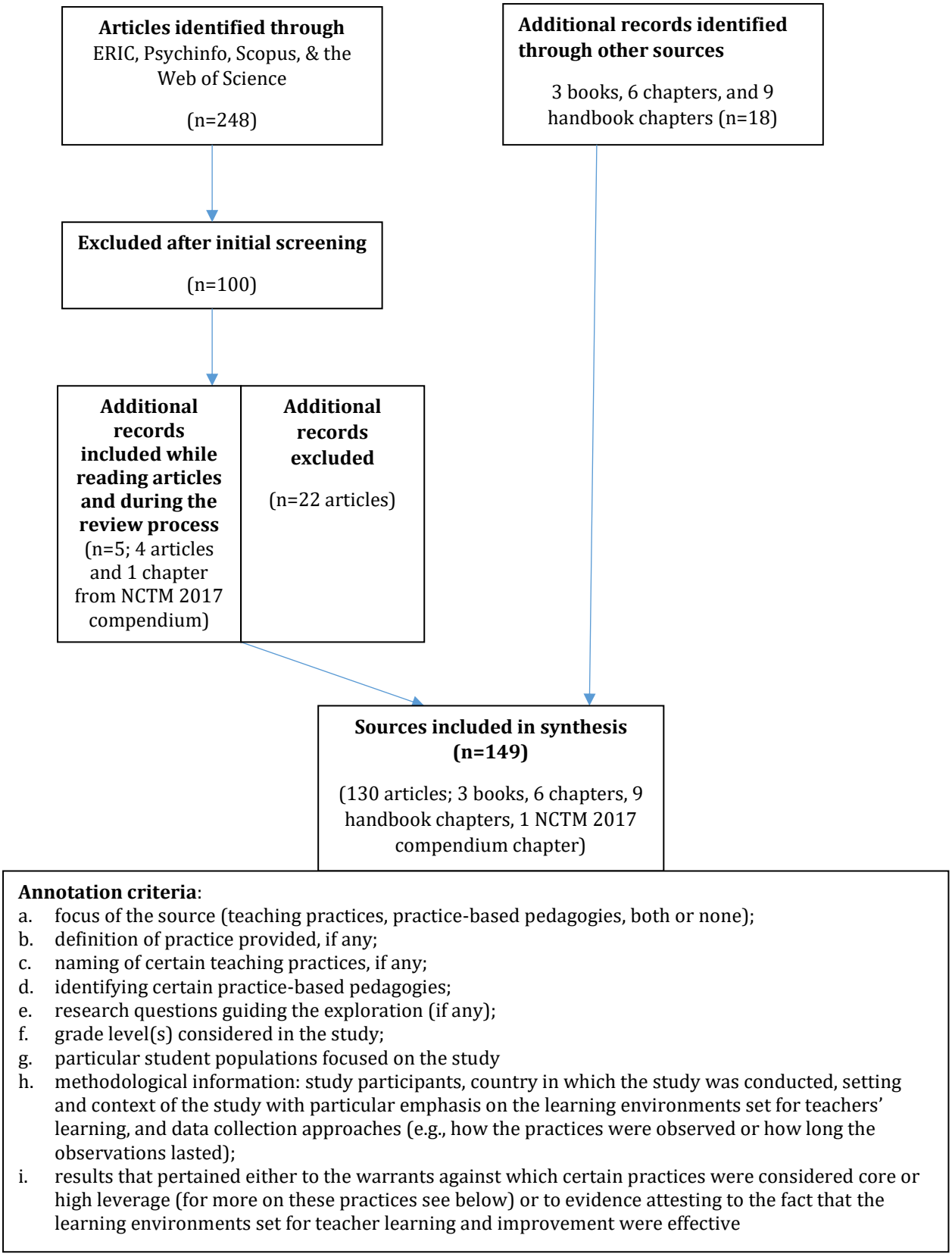

Figure 1. *Flowchart of the literature search and annotation*

Next, articles identified were screened. We worked independently, reviewing each abstract and justifying decisions for inclusion or exclusion. Sources for which there was agreement on their unsuitability were dropped; at least one vote for a source merited its inclusion. Excluded were papers focused primarily on policy issues, teachers' beliefs about practice, teachers' knowledge and motivation about teaching, measurement approaches in studying instructional quality, and the role of technological advancements (e.g., whiteboards) in improving instructional quality. We also excluded documents without an explicit focus on mathematics and its teaching or generally if its focus was tertiary education, thus limiting our review to pre-primary, primary, and secondary education.[3] This initial screening resulted in dropping 100 articles.

Third, the identified resources were read and annotated. Each document was entered on an Excel spreadsheet, recording the focal points that appear at the bottom of Figure 1. Following this step, 22 additional documents not meeting the criteria for inclusion were excluded. In addition, four articles cited in the initial documents but not identified in our search were included. As a result, our literature synthesis is based on 149 documents (marked with an asterisk in the reference list[4]).

Fourth, we read all articles and independently identified initial themes under two categories: (i) Practices of teaching and (ii) Practice-based pedagogies in teacher education (or combinations thereof). These initial themes were compared and common themes were identified, clarified and refined. Where themes emerged that were not common, agreement was sought on their inclusion or exclusion through discussion. Common themes were chosen which were deemed to represent either advances made in the fields under exploration or to suggest (explicitly or implicitly) open issues and areas where additional work is needed. Having identified themes, we next sought similarities and differences in the literature (e.g., what teaching practices have been identified or what practice-based pedagogies have been studied so far; what evidence exists about their contribution to either improving teaching quality and/or student learning). Study features such as the teacher population, the context and the research methods used further informed the chosen themes. Below, we present the themes that emerged from this analysis organized into two main sections, one corresponding to practices of teaching and the second pertaining to practice-based pedagogies in teacher education.

Attending to Practices Entailed in Teaching: Making Practice a Central Issue of Inquiry

This section is organized in five parts. First, we focus on issues around defining and understanding practice(s), pointing both to the different ways in which this term has been conceptualized, but also to the lack of shared language around defining this concept. Next, we zoom in on scholarly work around defining and classifying core or high leverage practices; third, we discuss the work of decomposing teaching to identify such practices. Fourth, we attend to empirical evidence generated about how core or high leverage practices contribute to student learning. In the fifth part, we consider how practices (in general) have been classified and measured/assessed. In the final part, we

[3] During the third stage of our search, we retained a couple of articles on tertiary education which were deemed to contribute something innovative or illustrative to the field.

[4] In the reference list we also present six chapters in Grossman's edited 2018 book separately.

examine how issues related to equity and culture are interwoven in the discourse around teaching practices.

Defining "Practice"

The term "practice" has been widely used in recent mathematics education literature. But the term has many meanings, little consistency exists in how it is used, and explicit definitions rarely accompany the term. One might infer that the meaning is implicitly clear or that it can be deduced from the context in which the term is used. Or perhaps the definition is elusive because its meaning is specific to contexts and "the meaning of a practice comes from its *use* [italics in original] in a community and the value of that practice in that community" (Staples, Bartlo & Thanheiser, 2012, p. 461). Given the amount of literature generated around the term and its centrality to analyzing teaching and organizing teacher education, more consistency in use of the term would likely help advance this body of research.

Lampert (2010) attempted to "provoke" clarity (p. 21) in the field by investigating four conceptions of practice. One conception of practice is that it involves implementing an idea in a context and is different to having the idea; this is the commonly made distinction between theory and practice. A second conception of practice is something that is done repeatedly in order to improve performance in it, in the sense of how one might practice playing tennis or the piano. Third, the practice of teaching includes people who have adopted "the identity of a teacher" who have been "accepted as a teacher" and who have taken "on the common values, language, and tools of teaching" (p. 29). This meaning is often used in the context of a medicine or engineering practice.

A fourth meaning of practice, which is typically used in the plural form, practices, relates to routines that are done "constantly and habitually" (p. 25) in the classroom. Other scholars complement this definition, by pointing out that practices are performed by "taking into consideration teachers' working context, and their meanings and intentions" (Maryono, Sutawidjaja, Subanji, & Irawati (2017, p. 12) and that they require both professional judgment and the involvement of "meaningful intellectual and social community for teachers, teacher educators, and students" (McDonald, Kazemi, & Kavanagh, 2013, p. 378); we revisit these, themes later in the chapter. An example of such a practice is orchestrating group discussions (Hatch & Grossman, 2009). It is this fourth definition of practice that has become a topic of research interest for researchers as part of a renewed interest in practice in teacher education (Zeichner, 2012).

Notwithstanding attempts by Lampert and others to bring clarity to the field, several examples exist of multiple meanings of the term, sometimes even within a single sentence, like noting that "successful enactment of these practices are [sic] typically found only in isolated pockets of practice" (Rosenquist, Henrick & Smith, 2015, p. 43). Elsewhere, Lerman and Zehetmeier (2008) use the term in several contexts: "researching practice", "communities of practicing mathematics teachers", "reflective practice" "how to organize a practice community", "the time for teacher talk and student practice went down", "they often lack knowledge and practice regarding these new issues", and "the relationship of theory and practice" (see also Lai, Auhl, & Hastings, 2015; Lloyd, 2013; Mayrowetz, 2009). Such examples show how the term can be used as

a noun, a verb or an adjective and the meanings can include performance, to be actively engaged in a career, a way of learning (through reflection or interaction), to perform repeatedly, experience, and apply.

In addition to multiple definitions of practice, a widespread reluctance to define the term and lack of agreement among definitions of practice, the muddiness is compounded by the use of other terms as quasi-synonyms for one or other meaning of practice. Such terms include procedure (Maccini & Gagnon, 2006; Lloyd, 2013), technique (Maher, 2008), strategy (Merritt, Palacios, Banse, Rimm-Kaufman, & Leis, 2017), and instructional decision (Shechtman, Roschelle, Haertel & Knudsen, 2010). The term "task" has been used both as a synonym for practice (Russ, Sherin, & Sherin, 2015) and as a sub-component of a practice (Sleep, 2012).

The meaning of practice used will depend on an author's theoretical perspective. Our intention is not to impose a definition of practice but to advocate care in articulating what meaning of the term applies and the theoretical perspective which underlies it. In this chapter we define practice as regular and habitual classroom routines engaged in by the teacher (Lampert, 2010, fourth definition) in a particular community, requiring professional judgment (McDonald et al, 2013). Specifically, we refer to practices known as "core" or "high leverage."

Focusing on Core or High Leverage Practices

Defining Practices as Core or High Leverage. One area on which researchers have intensively focused is on identifying practices that are generative (Lampert 2010), salient (Lampert et al., 2013), fundamental (Santagata, 2005), foundational (Shaughnessy & Boerst, 2018), core (McDonald et al, 2013; Shaughnessy & Boerst, 2018) or high leverage (Sleep & Boerst, 2012). The latter two terms are widely used, often interchangeably even though nuanced differences exist – "core" refers to teaching practices generally whereas "high leverage" focuses on practices that are worthwhile for prospective teachers to learn.

Core practices have been defined as "identifiable components of teaching that teachers enact to support learning. These components include instructional strategies, and the subcomponents of routines and moves. Core practices include general and content-specific practices" (Grossman, 2018, p. 184). General examples include "implementing norms and routines for classroom discourse and work" (p. 165) "eliciting and responding to student thinking" (p. 171)," (p. 165) and "providing feedback to students" (p. 179). Subject-specific practices are, for example, "constructing and interpreting models" in science (p. 179) and "selecting and adapting historical sources", in history (p. 181). The definition, which was developed by the Core Practice Consortium[5] and likely involved compromise and accommodation among group members, provides little guidance for how a practice might be identified as core. For example, would instructional strategies such as questioning, planning a lesson, or valuing diverse voices and perspectives be included? Would they be classified as practices or subcomponents

[5] This is a U.S.-based multi-institution, multi-disciplinary group of teacher educators who aspire to achieve shared understandings around practices of teaching. See more at https://www.corepracticeconsortium.com/.

of practices? Ambiguity exists about the relationship between practices and subcomponents of practice, routines and moves.

More specificity is provided by McDonald and her colleagues (2013) who cite the work of Grossmann, Hammerness et al. (2009) and identify criteria that core practices might share:

> they occur with high frequency, they permit beginning mastery by novices, they can be enacted by novices across different curricula or instructional approaches, they allow novices to learn more about students and about teaching, they preserve the integrity and complexity of teaching, and they are research-based and have the potential to improve student achievement. (p. 277)

High leverage practices are conceived with an explicit focus on supporting prospective teachers in learning to teach:

> [T]hose practices at the heart of the work of teaching that are most likely to affect student learning… [they] comprise the essential activities of teaching; if teachers are unable to discharge them competently, they are likely to face significant problems. Competent enactment of such practices also lays the foundation for beginning teachers to develop into highly effective professionals (Ball & Forzani, 2010/11, p. 43).

Among sample practices identified are eliciting and responding to students' ideas (Lampert et al., 2013; McDonald et al., 2013; Shaughnessy & Boerst, 2018; Sleep & Boerst, 2012), organizing a mathematical discussion (Kosko & Wilkins, 2015; Tyminski, Zambak, Drake, & Land, 2014), and providing instructional explanations (Kline & Ishii, 2008; Russ et al., 2016).

Classifying Practices as Core or High leverage. For practices to be classified as core or high leverage, they must be evidence-based or research-based (e.g. McDonald et al., 2013). Some studies attempted to link the implementation of specific practices to student outcomes (see more on this below). For example, Webb and colleagues (2017) found that teacher support of student participation influenced student participation which, in turn, influenced student achievement. Lerkkanen and colleagues (2012) compared how child-centered practices and teacher-directed practices affected kindergarten children's interest in reading. Greater interest was associated with experiencing more child-centered practices and fewer teacher-directed practices. Although practices observed in classrooms may be supported by research, that is not universally true. Xenofontos (2016) found that some of the practices used by teachers are not always optimal. One reported practice was lowering of expectations for what immigrant students can achieve in mathematics class. The author criticizes this, observing that the practice removes responsibility from the teacher.

Despite the definitions and criteria, and consensus on a handful of practices, identifying definitive core or high leverage practices of teaching remains elusive. The lack of consensus is evident in how practices are classified, how components of practices are related, how lists of practices differ and in justifications for classifying them as core or high leverage. Given that the criteria and definitions proposed above are relatively

recent, it is not unusual to witness variation in how the concept has been interpreted by researchers working with different lenses, priorities, commitments, and experiences. Nevertheless, identifying disagreements helps recognize where potential for consensus exists and where differences are substantive. Resolving such differences will help move the field towards Lortie's aspiration of a "common technical vocabulary" (1975, p. 73), a goal of the U.S. Core Practice Consortium (Grossman, Kavanagh, & Dean, 2018).

Decomposing Practice(s)

Core and high leverage practices are typically identified by decomposing the work of teaching. Decisions need to be made about the scope of a practice; if naming a practice is to be helpful, it cannot be too big or too small. Subsequently, practices themselves may be further decomposed either for teacher education purposes or to map the terrain more precisely. We first review studies that attempted to decompose practice(s) before making suggestions for advancing such work in future.

Sleep and Boerst (2012) envisage domains of teaching that contain practices, and practices of teaching which contain techniques that can be "specified, taught to, and worked on by beginners" (p. 1039). Although technique is not explicitly defined, an example is "checking whether correct answers are supported by correct reasoning" (p. 1039). "Assessing student thinking" is the example they give of one domain and practices and techniques nested within that domain are shown in Figure 2. Even this relatively straightforward decomposition of a domain of teaching, is decomposed in a way that includes "nested practices of varying grain sizes" (p. 1039), some of which are also known as subcomponents, and one technique.

Elsewhere Shaughnessy and Boerst (2018) classify eliciting, responding, and interpreting as three "interactive" (p. 42) practices whereas Teaching Works[6] combines two of them – eliciting and interpreting individual students' thinking – into one high leverage practice (Grossman, 2018). Shaughnessy and Boerst (2018) focus on the "foundational" practice of "eliciting student thinking" and they identified four core components of the practice that they expected preservice teachers to demonstrate: "(a) eliciting the student's process, (b) probing the student's understanding of key mathematical ideas, (c) attending to the student's ideas, and (d) deploying other moves that support learning about student thinking" (p. 45). Each component is associated with moves that would make visible or not that component of performance. Moves are "specific steps of talk that teachers take as they interact with students … or particular actions" (p. 45).

Elsewhere Sleep (2012) refers to tasks as subcomponents of "steering instruction toward the mathematical point." Although she classifies "steering instruction toward the mathematical point" as part of the "work of teaching" rather than a practice of teaching, her goal in the research is to decompose this aspect of the work of teaching and to identify subcomponents of it.

[6] See http://www.teachingworks.org/. Accessed on 1 December 2018.

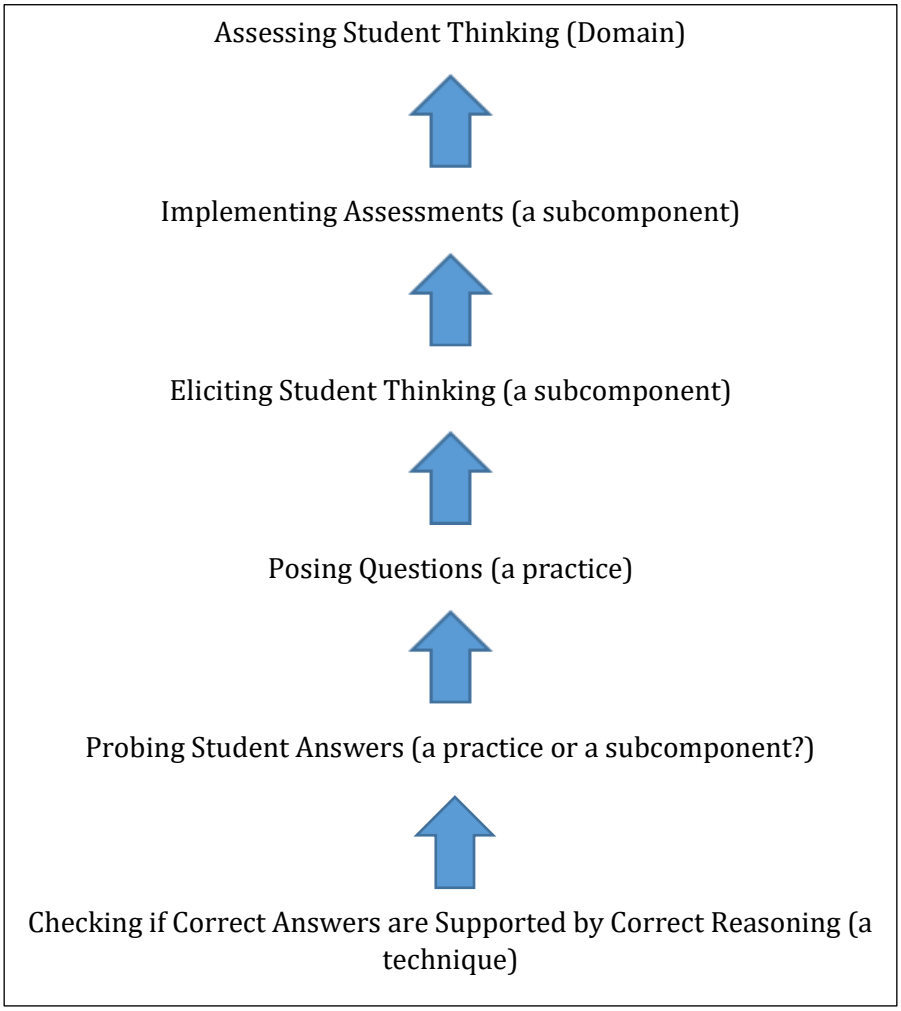

Figure 2. *How techniques are nested within practices of different grain sizes which are nested within the domain of "Assessing Student Thinking" (based on Sleep & Boerst, 2012).*

Similarly when Tyminski and colleagues (2014) decompose the practice of organizing a mathematical discussion, they draw on Smith and Stein's work (2011) as "one way to accomplish the decomposition of a complex practice" (p. 468) into the five practices of anticipating, monitoring, selecting, sequencing, and connecting. Tyminski et al. imply that practices are nested within other practices. Santagata and Yeh (2014) are also interested in instructional conversations, but they focus on how student thinking is made visible. Their decomposition of the practice is presented at three levels of sophistication: low (student thinking is minimally visible or not visible), medium (student thinking is visible) and high (student thinking is both made visible and probed further).

Briefly surveying how practices are decomposed illustrates the lack of consistency around language, definition, detail, and grain size. Comparing lists of practices articulated by different institutions and collaborators (see appendices in Grossman, 2018) confirms this finding. Such differences and lack of consistency may be symptomatic of a field in exploratory mode regarding practices of teaching and practice-based approaches to teacher education. Indeed, if consensus is to be achieved, it will likely take time. Yet, given the green shoots of research currently visible, researchers in

the field could strive for more consistency when studying similar practice-related ideas. Little or slow progress seems likely unless the fruits of collaboration among practice-focused research groups in the United States (Grossman, Kavanagh, & Dean, 2018), become more evident in that country initially, because much of the impetus for this work is sourced there. Building a common technical vocabulary among researchers and teacher educators across and beyond the United States poses additional challenges.

Although decomposition is necessary for understanding the work of teaching and making it teachable and learnable, focus must remain on the bigger picture and the complexity of teaching. Therefore, we encourage more researchers to attend to both aspects—decomposition and recomposition of practices (Sleep, 2012)—showing how prospective and practicing teachers can be helped to study and learn individual practices, and the practice of teaching in general.

Empirical Evidence on the Contribution of High leverage or Core Practices to Student Learning

Most definitions of core or high leverage practices aspire to improve student learning. Ball and colleagues (2009) claim that "proficient enactment of [such practices] by a teacher is likely to lead to comparatively large advances in student learning" (p. 460). The Core Practice Consortium identifies practices that constitute "strategies, routines, and moves that can be unpacked and learned by teachers to support student learning" (cited in Grossman, et al., 2018, p. 4), while Grossman, Hammerness, & McDonald (2009) argued that these practices are research-based and can improve student achievement. We now consider empirical evidence that these practices support student learning, purportedly through improving teaching quality.

Defining practices remains a problem in reviewing such evidence. First, studies that explore how practices of teaching contribute to student learning do not necessarily identify these practices as such—let alone call them core or high leverage; instead they refer more generally to different instructional aspects or teaching factors. Second, several other studies implicitly or more explicitly cluster individual strategies or teacher techniques under the term "teaching practice(s)" without, however, having the features that teaching practices, as defined above, are expected to have. Acknowledging such complications, the brief literature review that follows aims to highlight certain topics instead of providing a comprehensive account of what has been accomplished in the field.

Interestingly, few studies provide empirical evidence supporting the contribution of these practices to student learning. Most studies that provide such evidence were published during the last decade and pertain to kindergarten and primary-school grades (e.g., Bargagliotti, Gottfried, & Guarino, 2017; Blazar, 2015; Bottia, Moller, Michelson, & Stearns, 2014; Cohen, 2018; Firmender, Gavin, & McCoach, 2014; Ing et al., 2015); much rarer are studies focusing on secondary grades (e.g., Charalambous & Kyriakides, 2017; Fyfe & Rittle-Jonhson, 2017). Scholars seem to have followed different data-collection approaches when examining teaching practices, ranging from live or videotaped classroom observations (e.g., Blazar, 2015; Cohen, 2018 Firmender et al., 2014) to teacher self-reports (e.g., Bargagliotti et al., 2017; Bottia et al., 2014), and student surveys (e.g., Charalambous & Kyriakides, 2017). Although pursuing a suite of methodological approaches in studying this effect is both desirable and necessary, given the complexity of teaching, scholars are rarely explicit about the affordances and

limitations of their chosen evaluative approach(es) and the decisions that guided their selections. Explicitness around such matters is critical for making more compelling arguments about incorporating the practices in initial and in-service teacher education (see similar arguments in Praetorius & Charalambous, 2018).

Although some negative or non-significant effects have been identified (e.g., Bargalliotti et al., 2017), most studies reviewed provide encouraging results suggesting that different practices such as teacher modeling (Cohen, 2018), providing feedback (Fyfe & Rittle-Johnson, 2017), questioning and structuring (Charalambous & Kyriakides, 2017), practices encouraging productive dialogue (Webb et al., 2017) or immersing students in rich and cognitively challenging learning environments (Blazar, 2015) are positively associated with student achievement or learning. The promise of combining teaching practices that cut across different subject matters—often called generic practices—with mathematics-specific practices in explaining student learning has also been empirically suggested (Charalambous & Kyriakides, 2017). Charalambous & Kyriakides drew on data from the TIMSS 2011 and 2015 cycles and showed that a higher percentage of the unexplained variance in student learning could be explained when combining practices as opposed to when considering generic or content-specific practices in isolation. Interestingly, this percentage seemed to vary across different countries, thus calling for a deeper exploration of the mediating role that different contextual factors might have on the association. Pointing to a missing link between teaching practices and student learning, another recent study (Ing et al., 2015) examined the role of student participation in mediating the relationship between teacher support and student learning. The authors advocate attending to both teaching and student participation to understand how teaching can affect student learning.

Despite such work on studying associations of core or high leverage practices with student learning, significant outstanding matters require consideration. First, in line with other scholars (Ball & Forzani, 2009; Hlas & Hlas, 2012), more concerted efforts are needed to empirically validate the contribution of these practices to student learning. The evidence generated to date seems thin and unsystematic. This could be addressed by developing a common lexicon and framework for describing and analyzing instruction (cf. Grossman & McDonald, 2008) and seeking consensus on a set of core practices considered important for teaching in general and for teaching mathematics in particular. Achieving such a consensus would be a substantial achievement.

Should such a consensus be achieved, it would create the possibility of explicitly investigating how these teaching practices influence student learning. This is particularly important because without clearly articulating specific hypotheses around the mechanisms through which teaching practices can affect student learning, developing and implementing research designs that lend themselves to capturing these effects seems difficult. Along with Hlas and Hlas (2012), we believe that the scholarly community could agree on what constitutes empirical evidence in investigating the aforementioned association and on what methodological designs might better lend themselves to generating such evidence. Although classroom observations are often viewed as the gold standard in exploring instructional quality (cf. Douglas, 2009), multiple approaches may more effectively capture how instruction contributes to student learning. These methodological considerations also need to be informed by a broad conception of student learning that moves beyond the cognitive outcomes to also

include other forms of learning, such as affective and meta-cognitive outcomes (cf. Reynolds et al., 2016).

Ing and colleagues (2015) point to another critical issue. By overemphasizing the role of teaching practices and the opportunities teachers craft for student learning, as a research community we run the risk of missing a crucial link in the chain connecting teaching and student outcomes: how the students themselves respond to opportunities crafted for their learning. In fact, a recent synthesis of different classroom observation instruments (Praetorius & Charalambous, 2018) revealed that what teachers do or how they interact with their students is valued over closely attending to students' participation and the extent to which students' avail of opportunities to learn.

Finally, different contextual factors (e.g., available curriculum resources, instructional approaches followed, school-level or system-level factors) can have a mediating role on the association between teaching practices and student learning. One characteristic identified by Grossman, Hammerness et al. (2009) is that these practices can be enacted in classrooms *across* different curricula or instructional approaches. However, the impact that such curricula or instructional approaches can have on the effect of these teaching practices remains an open issue. Coupled with Charalambous and Kyriakides' (2017) findings suggesting differences across countries and educational systems on the effect of generic and content-specific practices on student learning, such explorations seem imperative.

Our review has so far been focused on core and high leverage practices. Because many studies focus on practices of teaching without identifying them as such, we now consider these studies briefly, while discussing issues of measuring/exploring them.

Classifying and Measuring "Practices" as Other Than Core or High leverage

How Practices are Classified. In addition to core and high leverage practices, authors categorize practices of teaching in multiple and sometimes nuanced ways. Several researchers refer to the idea of best practices, either on the basis of research warrants (e.g. Eddy, Converse & Wenderoth, 2015) or based on teachers' selection of artefacts to represent their teaching at its best (Silver, 2010). Lampert (2010) sees a problem with borrowing this term from the business world because reference to "best practice" raises the question of what goal the practice is best for achieving and what evidence supports a practice's designation as "best."

Frequently, practices are designated as "instructional" (e.g. Lee, Walkowiak, & Nietfeld, 2017; Silver, 2010; Swars, Smith, Smith & Carothers, 2018). Lloyd (2013) defines instructional practices as "those that necessitate critical thinking, reasoning, high levels of abstraction, and problem solving" (p. 107) and she distinguishes them from social/emotional and management practices. Coming with an interest in teaching mathematics to students with learning disabilities, Maccini & Gagnon (2006) define instructional practices as being "both empirically validated and recommended practices for teaching math" to students with learning difficulties or emotional and behavioral difficulties (p. 218) and the requirement for empirical validation is echoed in many articles.

Many authors contrast types of practice, like reform-based and traditional (e.g., McCaffrey et al., 2001; McClintock, O'Brien & Jiang, 2005; Suurtamm, Koch, & Arden,

2010), teacher-directed and student-centered (Morgan, Farcas & MacZuga, 2015), and high press and low press (a term taken from Kazemi & Stipek, 2001 by Webb et al., 2009). Culturally responsive practices have also been identified (e.g. Ukpokodu, 2011). Culturally responsive practices are derived from culturally relevant or culturally responsive pedagogy and propose that all students experience academic success, develop or maintain cultural competence, and develop critical consciousness through which the current social order can be challenged (Ladson-Billings, 1995). Cultural practices will be discussed further below.

A more specific kind of practice, responsive classroom, is studied by Ottmar, Rimm-Kaufman, Larsen and Berry (2015). Responsive classroom refers to "a set of principles and practices for integrating social and academic learning across the school day, creating classroom management processes well aligned with children's social and emotional needs, and fostering a caring and responsive environment for students" (p. 792).

Other researchers avoid using the term practice. Webb and colleagues (2017) use the term (teacher) "move" or "intervention" where others might use "practice." Examples include "help students formulate their own ideas and consider others' perspectives" (p. 3), "asking students probing and clarifying questions" (p. 3) and "'positioning' students as capable participants (p. 7).

Measuring and Assessing Practices of Teaching.[7] Several instruments have been used by researchers to study or identify practices of teaching, each with their own conception of practices of teaching. The instruments include the Standards-based Learning Environment Protocol developed by Tarr et al. (2008) to observe students, teachers, lessons and their interactions (used by Swars et al, 2018); the Classroom Observation and Analytic Protocol developed by Horizon Research (2000) which is an observational tool in which the observer judges the lesson according to design, implementation, content and culture before assessing the lesson's likely impact on student learning; the Classroom Practices Observational Measure (CPOM) developed by Abry and colleagues (2010); and the Early Childhood Classroom Observation Measure – ECCOM (see Lerkkanen et al, 2012). In this instrument, observers rate management, climate and instruction in classrooms under two broad headings of child-centered teaching practices and teacher-directed teaching practices.

Articles that document measures of teaching practices are typically explicit in identifying the practices, even if the practices sometimes consist of a single word and lack the detail or nuance contained in lists of core or high leverage practices. The Teaching Practices Inventory was designed for use on university mathematics and science courses and it measures the extent of practices rather than the quality of their implementation, something that is more difficult to measure (Wieman & Gilbert, 2014). It identifies two categories of practice: Practices that support learning (such as

[7] Here we focus only on the (classroom observation) instruments that refer explicitly to measuring teaching practices. Over the past two decades several other instruments have been developed to measure certain instructional aspects, without, however, identifying them as teaching practices. For a review of these instruments see Praetorius and Charalambous (2008).

knowledge organization, long-term memory and reducing cognitive load, motivation, practice, feedback, metacognition, and group learning) and practices that support teacher effectiveness (such as connecting with students' prior knowledge and beliefs, feedback to instructors on their effectiveness, such as mid-course evaluations or repeated feedback from students, and gaining relevant knowledge and skills). Another instrument for studying teaching practices in large university classes is the Practical Observation Rubric to Assess Active Learning – PORTAAL (Eddy et al., 2015). It measures teaching along the dimensions of practice, logic development, accountability and reducing apprehension.

Shaughnessy and Boerst (2018) documented an innovative approach to assessing practices. Faced with the challenge of fairly and consistently assessing the practice of prospective teachers when real children responded in various ways, the authors adapted an idea from medicine, the "standardized patient" and developed the idea of the "standardized student." When assessing the practice "eliciting student thinking," prospective teachers interacted with an adult who was scripted to respond to tasks in a way many students might respond, thus presenting consistent challenges for the teachers who were being assessed. Such an approach, though time-consuming was innovative; each prospective teacher received a similar challenge in a setting where they were required to act as teachers rather than write about teaching. Such approaches suggest that in the future, classroom observations may be complemented by other innovative ways to better capture teachers' capacity to implement such teaching practices.

We close this section by discussing attempts to interweave the exploration of teaching practices with issues of equity and culture. Albeit short, this discussion is deemed necessary since it documents scholarly awareness about the fact that teaching needs to serve students with diverse needs and capabilities and that teaching constitutes a cultural activity (Stigler & Hiebert, 1999).

Interweaving Issues of Equity and Culture in Exploring Teaching Practices

Practices to Promote Equity. Students may be considered marginalized in schools for many reasons (e.g. bilingual students, students of color, students with learning disabilities). Hence Gutiérrez (2002) provides a definition of equity practice. She proposes a reorientation for how practices should be viewed in relation to equity concerns, and she specifies questions to be addressed by researchers in this area. In particular, equity practice refers

> to the practice enacted between teachers, students, and mathematics that empowers students to (a) develop proficiency in dominant mathematics, (b) develop critical stances and new perspectives on the relationship between mathematics and society, and (c) contribute toward a positive relationship between mathematics, people and society in ways that erase inequities on this planet (p. 174).

Although this conception of equity practice expects students to become competent in dominant mathematics, it also envisages students adopting a critical stance towards

what has been called an understanding of mathematics as "an exclusively European product" (Dowling, 1998, p. 3). In this expression Dowling is referring to a eurocentric, elitist view of mathematics and mathematics education that is promulgated by school mathematics and school mathematical texts in particular, despite the contention in mathematics education literature that mathematics features centrally in all human cultures.

In a study of two classrooms where English language learners made high achievement gains, Merritt and her colleagues (2017) found that the teachers used multiple representations of concepts, they emphasized the building of mathematical vocabulary, both checked frequently for understanding and spent time analyzing students' errors. In both classrooms students spent relatively little time in small group discussions and the researchers were surprised that neither teacher connected the mathematics to students' everyday lives. Perhaps, they hypothesize, teachers were focusing on preparing students to do decontextualized tasks similar to those they will encounter on high-stakes tests organized by the district.

These findings largely echo those of an earlier, similarly small study of three teachers where the more successful teachers with high numbers of Latino students were those who moved through the lesson more slowly, alternating whole class time with individual/small group time, who built on student contributions when introducing new mathematical vocabulary, and who responded explicitly to errors (Zahner et al., 2012).

In an essay about supporting English language learners[8]' learning of mathematics, Moschkovich (2013) recommends four practices all based around language. Instead of focusing on accurate language or single words, teachers should focus on mathematical reasoning and mathematical practices; teachers should support students in engaging in complex mathematical language; and students' home languages should be seen as resources and not obstacles to their progress in mathematics. Jackson and Wilson (2012) reviewed mathematics education research from 1989 to 2011, to identify instructional practices that are particularly conducive to African American students' learning. A key finding of their work was that current research supports only the identification of broad principles for teaching these students and the authors advocate continued investigation of practices of teaching that, with other factors, may enhance student learning.

Several authors make recommendations about how students with learning disabilities can be supported in learning mathematics (e.g. Maccini & Gagnon, 2006; Mayrowetz, 2009; Spooner et al., 2017). Spooner, Saunders, Root, and Brosh (2017) look at an "evidence-based practice for teaching mathematical problem solving to students with learning disabilities" called schema-based instruction. This practice has four components: make the problem accessible (e.g. through interactive read-alouds of story problems or meaningful and motivating contexts); make the problem conceptually comprehensive by providing graphic organizers and by sequencing problems from

[8] The term "English Language Learners" refers to students whose first language is not English and includes those both with early and considerable proficiency in the language (Lacelle-Peterson & Rivera, 1994).

easier to more difficult types; solve the problem procedurally; and generalize in several ways. Mayrowetz (2009) looked specifically at the treatment of tasks in inclusive classrooms with high incidence of students with special needs. Although teachers helped students on a one-to-one basis, it was rarer for teachers to modify their instruction for students with disabilities. Documenting how practices are inclusive of all learners will greatly enhance research and practice in this area.

Practices of Teaching as Cultural Artefacts. Much has been written about the cultural nature of teaching (e.g. Stigler & Hiebert, 1999; Stylianides & Delaney, 2011) and the impact of culture is evident in the literature about practice. This phenomenon refers to both differences in teaching across cultures and to how mathematics teaching is largely influenced by the culture in which it is applied. We elaborate further on each of these in relation to the articles reviewed.

Santagata (2005) compares the mistake management sequence in Italy and the United States and finds that whereas US teachers tend to move on from mistakes quickly, Italian teachers more typically asked students to correct their errors, often with support from the teacher. Although Santagata does not call the "mistake management sequence" a practice, it might be deemed the product of a practice in countries where it is used or it may require the naming and identification of related, nuanced practices in line with priorities in a given country. Lan and colleagues (2009) studied classroom practices in China and the United States. They found that practices associated with higher performance in China (e.g. large-group instruction) are not inherently effective but that other factors are related to effective practice such as student behavior and how lessons are enacted. "Whole-class instruction" is not necessarily a practice in itself, and is certainly not a reliable term across cultures, because it is enacted differently across countries: promoting active engagement in China versus primarily lecture time in the United States.

Wager (2012) identified four practices in which teachers engaged to help students in schools with large numbers of ethnic minority students connect their everyday lives to school mathematics. First was using students' out of school experiences as contexts for word problems. Second was relating cultural activities to school mathematics (e.g. mathematics of nutrition; fractions and cooking). Third was encouraging students to use informal strategies— often different to those used in school—to solve problems such as purchasing food for the family. Fourth, the classroom was used as a site of culture (e.g. modelling a daily store, studying butterfly migration to Mexico and money from around the world).

When identifying practices of mathematics teaching, although overlap likely exists in practices from one country to another, some practices will be country- or culture-specific. Furthermore, practices that acknowledge and value the diverse backgrounds of all students as a foundation for fostering their mathematical achievement must be included when practices are identified.

Given the current widespread interest in practices of teaching, it may be timely for the field to strive to agree on a conception of what a practice of teaching is. This could lead to mapping teaching through its practices, the description of categories of practice –

such as “core” and “high leverage” – and their relationship with domains, tasks, techniques, and subcomponents of practice. The extent to which practices are general or subject specific could be explored, as could their grain size. Means of evaluating the impact of using classroom practices on student learning could also be identified. Practices need to be articulated in a way that include students with disabilities and members of ethnic or linguistic minorities. Some practices are likely to be specific to or different in particular countries or cultures. In order to avoid practices being viewed as thoughtless routines, the exercise of teacher judgment in practices needs to be studied and highlighted in response to how McDonald et al. (2013) include this as a criterion. Similarly, the community-basis for adopting, implementing, and evaluating practices merits further study.

Practice-Based Pedagogies: Making Practice a Central Source of Teacher Learning

In this section, we focus on pedagogies that have the potential to foster teacher learning and which put teaching practice at the core. We first explain what is meant by practice-based pedagogies and we justify our decision to focus on representations decomposition, and approximations of practice, as an overarching umbrella covering such practice-based pedagogies. We then provide specific examples of implementing these pedagogies, and synthesize literature that explores their effectiveness in promoting teacher learning. We conclude by identifying open issues that warrant consideration as research on practice-based pedagogies accumulates.

Practice-Based Pedagogies: Focusing on Representations, Decomposition, and Approximations of Practice

The terms practice-based pedagogy or practice-based teacher education have been used in different ways to capture different approaches in teacher education (cf. Forzani, 2014). Despite the differences in use, the term mostly emphasizes the importance of engaging prospective and practicing teachers not in theoretical discussions around practice, but in enacting practice as a means of learning to teach. Realizing the limitations of traditional approaches in helping prospective teachers and novice practicing teachers learn to *do* (rather than to think about) the complex work of teaching (Forzani, 2014; Grossman, Kavanagh, & Dean, 2018; Grossman & McDonald, 2008; McDonald, Kazemi, & Kavanagh, 2013), the field of teacher education has recently witnessed a shift, turning away from an intense focus on the knowledge needed for teaching to the use of this knowledge in practice. As Grossman and colleagues (2018) suggest, this increased emphasis on practice was preceded by similar attempts in the first half of the previous century (e.g., the Commonwealth Teacher Training Study, see Forzani, 2014) and later in the 1960s and 1970s (competency-based teacher education), which led to long lists of discrete skills that prospective teachers and novice practicing teachers were expected to learn and practice. This reductive conception of teaching shifted the pendulum to the end of teacher knowledge and judgment, from which the field seems to gradually be departing (cf. Ball & Forzani, 2009; Gitomer & Zisk, 2015). Examples of such shifts can be seen in the restructuring of the teacher education programs, in the United States (see, for example, Ball, Sleep, Boerst, & Bass, 2009; Grossman et al., 2018), but also outside the United States [see, for example, Jao, Wiseman, Kobiela, Gonsalves, & Savard (2018) for analogous efforts in Canada].

Comparing professional pedagogies across three professions, Grossman and colleagues (2009) developed a framework for the teaching of practice that includes three elements: representations, decomposition, and approximations of practice. We now explain each component and then explicate why we regard this framework as useful in capturing different practice-based pedagogies.

Representations of practice are how practices of teaching are made visible to prospective, novice or experienced teachers. Possible representations include records of practice like videos of teaching, observations of live teaching, public lessons, modeling teaching practices, classroom transcripts, lesson plans, student artifacts, case studies about teaching, teacher narratives about teaching, and multimedia and animated portrayals of teaching (Ball, Ben-Peretz, & Cohen, 2014; Danielson, Shaughnessy, & Jay, 2018; Grossman et al., 2009; Han & Paine, 2010; Herbst & Kosko, 2014; McGrew, Aston, & Fogo, 2018). Different representations illuminate certain aspects of teaching, while obscuring others. Therefore teacher educators must carefully select representations to use, based on the purpose they are expected to serve (cf. Lampert & Ball, 1998).

Decomposition of practice refers to partitioning the complex work of teaching and its practices into identifiable, constituent parts. Teaching can be decomposed to identify core or high leverage practices as detailed above and decomposition can be performed at varying grain sizes. Decomposing practice allows for targeted instruction and scaffolding to help teachers learn and improve aspects of teaching. As Grossman and colleagues (2009) note, "By decomposing complex practices, professional educators can help [prospective teachers] learn first to attend to, and then to enact, the essential elements of practice" (p. 2069). In essence, decomposition is useful for developing professional vision (cf. Goodwin, 1994), and for enacting practice in environments that support experimentation with and reflection upon different components of teaching.

Approximations of practice, the third pillar of the framework, refer to creating safe environments for prospective or novice teachers to practice teaching with high degrees of support. By reducing complexity and focusing teachers' attention on specific aspects of teaching, approximations create productive spaces for experimentation and deliberate and substantive reflection on teaching. Equally important, they "reduce the error in the field", while still helping teachers to focus on "high-stakes practices" (Grossman et al., 2009; p. 2091). Approximations are used widely in other professions like medicine, dentistry, law, and pilot training (Schutz, Grossman, & Shaughnessy, 2018). In teacher education they can take different forms, including role plays (Schutz et al., 2018), replays (Horn, 2010), rehearsals (Ghousseini, 2017; Horn, 2010; Kelley-Petersen, Davis, Ghousseini, Kloser, & Monte-Sano, 2018), microteaching (Cheng, 2017; Hong & Chai, 2017; Lai et al., 2015), simulations of practice (Charalambous, 2008), field placement teaching, fishbowls, co-teaching, and processing pauses (Schulz et al., 2018) . These forms differ in their level of authenticity.

Admittedly this framework cannot encompass all different forms of practice-based pedagogies. For example, although it does create a space for teacher collaboration and learning, it does not necessarily capture the complexity of such collaborations, as manifested in communities of practice (e.g., Jaworski, 2006; Sowder, 2007). However, it can serve as an umbrella for different other professional development approaches, some of which have become particularly popular in the last two decades, such as lesson study (see Huang & Shimizu, 2016 for a systematic review) and video viewing in general

(see Gaudin & Chaliès, 2015 for a review) or video clubs, in particular (e.g., Charalambous, Philippou, & Olympiou, 2018; Russ et al., 2015; Sun & van Es, 2015).[9] Hence, although Grossman and colleagues' (2009) framework cannot comprehensively capture existing work on practice-based pedagogies, it offers a good heuristic for capturing three components of such pedagogies that are present to a greater or lesser extent in pedagogies that render practice and its enactment at the core of teacher professional development.

Examples of Incorporating Practice-Based Pedagogies in Teacher Education Programs

Over the last decade teacher education programs have introduced or reported on practice-based pedagogies. Some efforts are programmatic, involving restructuring an entire teacher education program; others involve revamping individual courses (cf. Cantun, Schutz, Kelley-Petersen, & Franke, 2018). Common to all such efforts is the identification of core or high leverage practices, which are represented in various ways, decomposed systematically, and approximated in controlled settings.

A programmatic level example is the systematic redesigning of the undergraduate program at the University of Michigan to center it more on practice (see TeachingWorks, http://www.teachingworks.org). Faculty from the University have collaborated with practicing teachers and graduate students to identify a set of high leverage practices that cut across different subjects. Following these year-long efforts, 19 such practices were generated, including leading a group discussion; eliciting and interpreting student thinking; specifying and reinforcing productive student behavior; and setting long and short-term learning goals for students (see Grossman, 2018, pp. 164-169 for a complete list). Following this decomposition of teaching, scholars designed courses to immerse prospective teachers in analyzing representations of teaching (as exemplified below) and in enacting certain elements of these practices in a heavily scaffolded environment. A similar endeavor was undertaken under the University of Washington *Accelerated Certification for Teachers* (U-ACT) initiative, which resulted in restructuring two graduate-level teacher education programs around seven core teaching practices, including orienting students to each other ideas, orienting students to the content, and assessing student understanding (see Grossman, 2018, pp. 170-173 for a list of these practices).

At course level, several initiatives have been undertaken by individual or small groups of faculty members, often within the same disciplinary area (for such examples see Cartun et al., 2018, pp. 109-133). Our review yielded several examples of how teacher educators have capitalized on the idea of representations, decomposition, and approximations of practice to restructure their teaching programs (e.g., Ball et al., 2009; Boerst, Sleep, Ball, & Bass, 2011; Erickson & Herbst, 2018; Ghousseini, 2015; Ghousseini & Herbst, 2016; Ghousseini, 2017; Herbst, Chieu, & Rougée, 2014; Horn, 2010; Shaughnessy & Boerst, 2018; Tyminski, Zambak, Drake, & Land, 2014). We now review two such examples which illustrate both the efforts invested in considering and structuring such environments, and the several decisions involved in capitalizing on this

[9] For example, lesson study can be considered an authentic approximation of teaching, since it occurs in actual classrooms and it preserves the complexity of the work; at the same time, the videotaped or live classroom observations that take part in the context of lesson study provide representations of the work of teaching, which can then be decomposed according to the foci set in each lesson-study session.

framework to inform the work. We highlight these examples because they explain clearly how Grossman et al.'s (2009) framework was used, something that seems to be the exception rather than the rule.

The first example pertains to a mathematics methods course for prospective elementary schoolteachers designed at the University of Michigan (Boerst et al., 2011). This course was revamped to emphasize four teaching practices: leading a classroom discussion; planning mathematics lessons; assessing students' knowledge, skill, and dispositions; and representing mathematical ideas. In the study the authors focus primarily on leading a classroom discussion and provide a detailed account of how they decomposed this practice into five identifiable components. This decomposition enabled the teacher educators to design various opportunities to engage prospective teachers in gradually learning this practice through approximations of practice. Specifically, prospective teachers were initially supported to engage in asking purposeful questions; then, to teach a "mini-problem" during field placement, and finally, to teach an entire mathematics lesson. This nesting of earlier approximations into subsequent ones and the gradual increase in complexity and authenticity allowed prospective teachers to progressively immerse themselves in the focal practice. Prospective teachers' work was scaffolded by representations of practice (e.g., videos of practice). In viewing these representations, prospective teachers were provided with viewing lenses that enabled them to focus their attention on specific aspects of teaching, starting narrowly, then expanding the lens gradually to include more aspects of the relevant practice.

The second example pertains to a mathematics methods course intended for prospective secondary school teachers (Ghousseini & Herbst, 2016). Although again focusing on leading a classroom discussion, the teacher educators in this study followed a different approach to that described above, thus illustrating another way in which teacher educators have applied Grossman et al.'s framework of practice-based pedagogies. The teacher educators began by representing a classroom mathematics discussion in which the prospective teachers worked as students on a warm-up problem and then discussed it as a whole class. One teacher educator deliberately modeled some of the work involved in leading a discussion by enacting instructional moves he wanted the prospective teachers to learn. This created a productive space to decompose work entailed in leading a classroom discussion, something that was done in collaboration with the prospective teachers themselves, who were asked to collectively label and elaborate aspects of this work. Like in the previous study, this was followed by different types of approximations of practice. First, the prospective teachers were given a constructed dialogue between an imaginary teacher and her students around a mathematical problem and were asked to complete the teacher lines in this dialogue which had been purposefully erased. Next the prospective teachers' rehearsed leading a classroom discussion in a fishbowl setting. In the third approximation, prospective teachers planned with their cooperating teachers to lead a classroom mathematics discussion in their field placement.

These two examples illustrate the potential of such courses to support prospective teachers in learning the work of teaching. They also typify the careful work needed in developing such courses, given that different decisions need to be made at several junctures, both when planning and when enacting and reflecting upon such efforts. In what follows, we review the literature with respect to evidence accumulated so far about the effectiveness of such practice-based pedagogies.

Empirical Evidence on the Contribution of Practice-Based Pedagogies

In this part, we focus on studies that provide empirical evidence about the contribution of practice-based pedagogies. Literature that discusses these issues theoretically or documents the design of programs/courses (e.g., Ball et al., 2009; Boerst et al., 2011; Grossman et al., 2009) were not considered. In addition to considering the evidence generated in these studies, we discuss three other matters that help contextualize this evidence: the opportunities afforded to teachers for enacting and reflecting upon their (own) practice; the teaching practices considered; the participants engaged in these studies; and the evidence generated with respect to (prospective) teacher learning.

Our review pointed to a notable variation in what is reported regarding the contribution of practice-based pedagogies to teacher learning. Some studies (e.g., Han & Paine, 2010; Tyminski et al., 2014) go into detail either illustrating how teachers' work changed as a result of participating in the practice-based learning environments or discussing the mechanisms that seemed to facilitate participants' learning (e.g., Horn, 2010). Other studies (e.g., Averill et al., 2016; Ghousseini, 2017), however, either present prospective teachers' learning in broad strokes or largely remain silent on this issue. In general, no common methodological framework seems to exist for guiding scholarly explorations about what counts as evidence on the contribution of these environments to teacher learning, let alone how this evidence needs to be generated and reported. These issues represent open areas that ought to be considered in the next decade, especially since work around practice-based pedagogies seems likely to grow. Scholarly attempts need to produce stronger and more systematic validity evidence about the potential of these practice-based approaches in supporting (prospective) teacher learning.

Given the emphasis on practice-based pedagogies, it is unsurprising that many studies report extensively on environments that were crafted to support prospective teachers' opportunities to enact practices of teaching (e.g., Averill, Drake, Anderson, & Anthony, 2016; Ghousseini, 2015, 2017; Schutz et al., 2018; Tyminski et al., 2014). Such reporting is reasonable, given the new (or renewed) emphasis on practice-based pedagogies in the past decade and scholars drawing on it have attempted to explicitly describe how they capitalize on its affordances to support teacher learning. A critical component in this reporting is the opportunities afforded to prospective teachers to reflect upon their own teaching or that of their peers. In rehearsals, the key approximation of practice for which empirical evidence has been generated, such opportunities are made available through debriefs often accompanying the approximation, either with the prospective teacher engaged in the rehearsal or with observers (see, for example, Averill et al., Ghousseini, 2015, Schutz et al., 2018). Studies that mostly focused on representations of practice (e.g., Herbst & Kosko, 2014; Herbst et al., 2014) report on how these records of practice facilitated eliciting different teacher ideas and beliefs about teaching. Despite their significance, these latter studies provide a window into teachers' *thinking* rather than their actual experimentation with the work of teaching.

Another pattern emerging from this analysis concerns the teaching practices that have been the focus of recent scholarly efforts. Interestingly, most of the works that provide empirical evidence on practice-based pedagogies focus on just a single practice—something not surprising given the publishing space restrictions. What is interesting, however, is that most of these works focus on the practice of leading a classroom discussion (e.g., Averill et al., 2016; Ghousseini, 2015; Ghousseini & Herbst, 2016; Tyminski et al., 2014). The focus on this practice can be attributed partly to the fact that

it represents a key component in several lists of core or high leverage practices and partly because it has been significantly decomposed and elaborated on in the literature (see, for example, Stein, Engle, Smith, & Hughes, 2008; Smith & Stein, 2011); it is also considered core in attempts to offer students high-quality instruction that elicits and builds upon their thinking (cf. Jacobs & Spangler, 2017). The focus on leading classroom discussions implies that different manifestations of practice-based environments around this practice have been generated, which enables their comparability. At the same time, it represents a significant open issue for years to come, because examples of other teaching practices are needed, to illustrate how practice-based pedagogies can be implemented and to support teacher learning in other teaching practices.

Unsurprisingly, the participants engaged in these studies are exclusively prospective teachers. The number of the participants reported in the studies, however, varies. In some studies (e.g., Ghousseini, 2015) only a single teacher is considered; in other cases (e.g., Averill et al., 2016; Ghousseini, 2017; Tyminski et al., 2014) many more participants are presented. Each approach has its affordances and limitations: single case studies provide the opportunity to document in more detail how the prospective teacher interacted with the practice-based learning environment; what is left unattended, however, is whether and how this environment actually functioned for the rest of the participants. Studies that report on a large number of teachers unavoidably report on the average, thus leaving the differential effect of such environments on their participants open to further inquiry. Therefore, future work needs to strike a balance between foregrounding individual cases and foregrounding the entire teacher sample.

Additional Open Issues and Possible Future Developments

Our review of literature on practice-based pedagogies surfaced additional open issues. First, as already mentioned, although the framework considers learning to take place in groups of teachers, more emphasis needs to be given to how these practice-based pedagogies can support communities of practice (see, for example, Sowder, 2007, for a discussion of such communities). Second, most of the extant studies apply the ideas of practice-based pedagogies within a U.S. context. Although this is reasonable, given that this is the context where the framework has been developed and given the current emphasis on practice-based pedagogies in the United States, future studies ought to test the applicability and transferability of these ideas in more countries and in different educational systems. Such explorations will allow for examining the extent to which certain contextual factors might impact on teacher learning; cultural adaptations of these environments and the reasons that lead to them will be particularly interesting to explore. Third, the matter of scaling needs to be examined. What has been reported so far mostly concerns attempts undertaken at small scale pertaining to single courses or even parts of these courses; as already mentioned, often times, the impact of these approaches is reported only for a few teachers and for individual teaching practices. It remains open to investigate the effect of larger-scale attempts (including more courses, more practices, and more, as well, as more diverse teacher participants) on teacher learning. Scaling up this effort will allow investigating of what is feasible resource-wise, in terms of time, personnel, course/program structures, and budget. Finally, given the over-emphasis placed on prospective teachers, a call is made for exploring the role of such environments in supporting the learning of novice practicing teachers or even practicing teachers at different stages in their career.

In the next section, we briefly consider both areas—teaching practices and practice-based pedagogies—and identify additional challenges and open issues for future consideration.

Working at the Confluence of Teaching Practices and Practice-Based Pedagogy: Other Challenges Warranting Consideration in the Future

Reflecting on the call to develop a practice-based pedagogy of teacher education, Peercy and Troyan (2017) identify additional challenges, besides those listed above that need to be borne in mind. Although coming from a different discipline (English Language Arts), the challenges these scholars outline can inform mathematics education as well. In particular, they recommend more transparency in how core practices are identified and developed. Given that lists of such practices in mathematics education are fast accumulating, as discussed above, this challenge needs careful consideration. The second challenge is that rarely is the complexity inherent in designing and implementing practice-based pedagogies made explicit. Hence, these scholars contend, "Specificity and transparency regarding the enactment of practice-based pedagogy across a number of disciplines would aid [teacher educators] in designing teacher education with core practices as an organizing framework" (p. 34). Given the increased emphasis on practice, these scholars warn that these practices might be divorced from their theoretical underpinnings. They thus suggest helping prospective and practicing teachers to focus on the work of teaching without, however, losing sight of the theoretical background supporting certain practices. Another challenge relates to combining practices of teaching with other aspects with which beginning teachers need to be concerned such as textbooks as well as curriculum and behavior management. Lampert and her colleagues (2013) advise that "practices, principles, and mathematical knowledge must be used *in relation to one another* [italics in original] not in isolation" (p. 228).

Finally, we are mindful that attempts to achieve more consistent use of language in relation to practices of teaching and practice-based pedagogies are limited by current economic and political circumstances. Teacher educators are constrained by what is possible through budgets and university structures. Research teams are constrained by research commitments which may be inconsistent with selfless collaboration (see Zeichner, 2012 for a discussion of such constraints).

Conclusion

This study reviewed literature on two exciting lines of research in mathematics education: teaching practices and practice-based pedagogies. The review suggests that 20 years after Ball and Cohen's (1999) call to focus on practice and start developing practice-based curricula, substantial scholarly advances have been made in this area, in both breadth and depth. In particular, with respect to the former strand, concerted efforts have been undertaken to decompose the work of teaching into identifiable practices of varying grain sizes; and some attempts have been made to study the effect of these practices on student learning. With respect to the latter strand, Grossman and colleagues' (2009) framework of representations, decomposition, and approximations of practice appears to have offered some common ground; the last decade has seen attempts to restructure teacher education programs by including elements of practice-based pedagogies. Despite the rapidly accumulating work in both strands, several issues

remain open: at a conceptual/theoretical level, at least more consistency in the language used is needed; at a methodological level, the absence of an agreed-upon suite of methodologies to empirically examine theoretical arguments advanced in these areas results in studies that do not build on each other to produce cumulative knowledge. We argue that stronger and more systematic empirical validation of the potential of teaching practices and practice-based approaches to teacher education is needed, something that appears to be one of the central open issues and challenges for the future.

The challenges and open issues identified in this chapter and the promise of the work produced to date underline the usefulness of continued, systematic, and more collaborative work around these issues in the years to come, both within the United States and internationally. In addition to continue decomposing teaching to identify practices of teaching and developing practice-based pedagogies—and finding productive ways of doing so— along with other scholars (e.g. Jabocs & Spangler, 2017) we argue that this work ought to be directed toward more systematically exploring how these teaching practices and their incorporation in practice-based pedagogies can improve instructional quality and through that the learning of *all* students. Bearing in mind the open issues and challenges, it seems that both strands of research examined here offer substantial opportunities for ongoing work, when examined in isolation, but more critically, when considered together.

References

* Akyuz, D., Dixon, J. K., & Stephan, M. (2013). Improving the quality of mathematics teaching with effective planning practices. *Teacher Development, 17*(1), 92-106.
* Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education, 9*(1), 33-52. http://dx.doi.org.ucd.idm.oclc.org/10.1007/s10857-006-9005-9
* Averill, R., Drake, M., Anderson, D., & Anthony, G. (2016). The use of questions within in-the-moment coaching in initial mathematics teacher education: Enhancing participation, reflection, and co-construction in rehearsals of practice. *Asia-Pacific Journal of Teacher Education*, *44*(5), 486-503. doi: 10.1080/1359866X.2016.1169503.
* Awang, H., & Ismail, N. A. (2006). Teaching and learning practices: Their effects on mathematics achievement. *Journal on School Educational Technology, 1*(3), 69-73.
* Ball, D. L. (2000). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education*, *51*(3), 241-247.
* Ball, D. L., Ben-Peretz, M., & Cohen, R. B. (2014). Records of practice and the development of collective professional knowledge. *British Journal of Educational Studies*, *62*(3), 317-335. doi: 10.1080/00071005.2014.959466.

Ball, D. L. & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In G. Sykes and L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3–32). San Francisco: Jossey Bass.

* Ball, D. L., & Forzani, F. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60*(5) 497–511. DOI: 10.1177/0022487109348479

Ball, D.L., & Forzani, F. (2010). Teaching skillful teaching. *Educational Leadership*, *68*(4), 40-45.
* Ball, D. L., Sleep, L., Boerst, T. A., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *The Elementary School Journal*, *109*(5), 458-474.
* Bargagliotti, A., Gottfried, M. A., & Guarino, C. M. (2017). Educating the whole child: Kindergarten mathematics instructional practices and students' academic and socioemotional development. *Teachers College Record, 119*(8), 1-41.
* Bass, H. (2017). Designing opportunities to learn mathematics theory-building practices. *Educational Studies in Mathematics*, *95*(3), 229-244. doi: 10.1007/s10649-016-9747-y.
* Biccard, P., & Wessels, D. (2017). Developing mathematisation practices in primary mathematics teaching through didactisation-based teacher development. *African Journal of Research in Mathematics, Science and Technology Education*, *21*(1), 61-73. doi: 10.1080/18117295.2017.1283184.
* Blazar, D. (2015). Effective teaching in elementary mathematics: Identifying classroom practices that support student achievement. *Economics of Education Review*, *48*, 16-29. doi: 0.1016/j.econedurev.2015.05.005.
* Boaler, J. (2008). When politics took the place of inquiry: A response to the National Mathematics Advisory Panel's review of instructional practices. *Educational Researcher*, *37*(9), 588-594. doi: 10.3102/0013189X08327998.
* Boerst, T. A., Sleep, L., Ball, D. L., & Bass, H. (2011). Preparing teachers to lead mathematics discussions. *Teachers College Record*, *113*(12), 2844-2877.
* Bottia, M. C., Moller, S., Mickelson, R. A., & Stearns, E. (2014). Foundations of mathematics achievement: Instructional practices and diverse kindergarten students. *The Elementary School Journal, 115*(1), 124-150. http://dx.doi.org.ucd.idm.oclc.org/10.1086/676950
* Boyd, D., Grossman, P., Hammerness, K., Lankford, H., Loeb, S., Ronfeldt, M., & Wyckoff, J. (2012). Recruiting effective math teachers: Evidence from New York City. *American Educational Research Journal*, *49*(6), 1008-1047. doi: 10.3102/0002831211434579.
* Cantun, A., Schutz, K. M., Kelley-Petersen, & Franke, M. (2018). Core practices and the teacher education curriculum: Stories of practice. In P. Grossman (Ed.), *Teaching core practices in teacher education* (pp. 107-133). Cambridge, MA: Harvard Education Press.
* Cavanagh, M., & Prescott, A. (2010). The growth of reflective practice among three beginning secondary mathematics teachers. *Asia-Pacific Journal of Teacher Education, 38*(2), 147-159. http://dx.doi.org.ucd.idm.oclc.org/10.1080/13598661003678968
Charalambous, C. Y. (2008). *Preservice teachers' Mathematical Knowledge for Teaching and their performance in selected teaching practices: Exploring a complex relationship*. Unpublished doctoral dissertation, University of Michigan, Ann Arbor.
* Charalambous, C. Y. (2016). Investigating the knowledge needed for teaching mathematics: An exploratory validation study focusing on teaching practices. *Journal of Teacher Education*, *67*(3), 220-237. doi: 10.1177/0022487116634168.
* Charalambous, C. Y., & Kyriakides, E. (2017). Working at the nexus of generic and content-specific teaching practices: An exploratory study based on TIMSS secondary analyses. *The Elementary School Journal, 117*(3), 423-454.

Charalambous, C. Y., Philippou, S., & Olympiou, G. (2018). Reconsidering the use of video clubs for student-teachers learning during field placement: Lessons drawn from a longitudinal multiple case study. *Teaching and Teacher Education, 74*, 49-61. https://doi.org/10.1016/j.tate.2018.04.002

* Charalambous, C. Y., & Pitta-Pantazi, D. (2016). Perspectives on priority mathematics Education: Unpacking and understanding a complex relationship linking teacher knowledge, teaching, and learning. In L. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (3rd ed., pp. 19-59). UK: Routledge.

* Cheng, J. (2017). Learning to attend to precision: The impact of micro-teaching guided by expert secondary mathematics teachers on pre-service teachers' teaching practice. *ZDM Mathematics Education*, *49*(2), 279-289. doi: 10.1007/s11858-017-0839-7.

* Chieu, V. M., Kosko, K. W., & Herbst, P. G. (2015). An analysis of evaluative comments in teachers' online discussions of representations of practice. *Journal of Teacher Education*, *66*(1), 35-50. doi: 10.1177/0022487114550203.

* Christman, J. B., Ebby, C. B., & Edmunds, K. A. (2016). Data use practices for improved mathematics teaching and learning: The importance of productive dissonance and recurring feedback cycles. *Teachers College Record, 118*(11), 1-32.

* Cohen, D. (2011). *Teaching and its predicaments.* Cambridge, MA: Harvard University Press.

* Cohen, J. (2018). Practices that cross disciplines?: Revisiting explicit instruction in elementary mathematics and English language arts. *Teaching and Teacher Education, 69*, 324-335. http://dx.doi.org.ucd.idm.oclc.org/10.1016/j.tate.2017.10.021

* Copur-Gencturk, Y., & Papakonstantinou, A. (2016). Sustainable changes in teacher practices: A longitudinal analysis of the classroom practices of high school mathematics teachers. *Journal of Mathematics Teacher Education, 19*(6), 575-594. http://dx.doi.org.ucd.idm.oclc.org/10.1007/s10857-015-9310-2

* Crisan, C., Lerman, S., & Winbourne, P. (2007). Mathematics and ICT: A framework for conceptualising secondary school mathematics teachers' classroom practices. *Technology, Pedagogy and Education, 16*(1), 21-39. http://dx.doi.org.ucd.idm.oclc.org/10.1080/14759390601167991

* Danielson, K. A., Shaughnessy, M., & Jay, L. P. (2018). Use of representations in teacher education. In P. Grossman (Ed.), *Teaching core practices in teacher education* (pp. 15-33). Cambridge, MA: Harvard Education Press.

* da Ponte, J. P., & Chapman, O. (2006). Mathematics teachers' knowledge and practices. In A. Gutiérrez, & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present, and future* (pp. 461-494). Rotterdam: Sense Publishers.

* da Ponte, J. P., & Chapman, O. (2016). Prospective mathematics teachers' learning and knowledge for teaching. In L. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (3rd ed., pp. 275-296). UK: Routledge.

* Delice, A., Aydın, E., & Çevik, K. S. (2013). Mathematics teachers' use of questions: Is there a change of practice after the curriculum change? *Eurasia Journal of Mathematics, Science & Technology Education, 9*(4), 417-427. http://dx.doi.org.ucd.idm.oclc.org/10.12973/eurasia.2013.9410a

* Di Muro, P. (2006). Best practices in mathematics instruction: Teaching for understanding. *NADE Digest, 2*(1), 1-8.

* Doabler, C. T., Nelson, N. J., Kennedy, P. C., Stoolmiller, M., Fien, H., Clarke, B., ... & Baker, S. K. (2018). Investigating the longitudinal effects of a core mathematics program on evidence-based teaching practices in mathematics. *Learning Disability Quarterly*, 1-15. doi: 10.1177/0731948718756040.
* Doabler, C. T., Nelson, N. J., Kosty, D. B., Fien, H., Baker, S. K., Smolkowski, K., & Clarke, B. (2014). Examining teachers' use of evidence-based practices during core mathematics instruction. *Assessment for Effective Intervention, 39*(2), 99-111. http://dx.doi.org.ucd.idm.oclc.org/10.1177/1534508413511848
Douglas, K. (2009). Sharpening our focus in measuring classroom instruction. *Educational Researcher, 38* (7), 518-521.
Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths /pedagogic texts.* Oxon: Routledge Falmer.
* Eddy, S. L., Converse, M., & Wenderoth, M. P. (2015). PORTAAL: A classroom observation tool assessing evidence-based teaching practices for active learning in large science, technology, engineering, and mathematics classes. CBE - Life Sciences Education, 14(2), 16.
* Erickson, A., & Herbst, P. (2018). Will teachers create opportunities for discussion when teaching proof in a geometry classroom? *International Journal of Science and Mathematics Education*, *16*(1), 167-181. doi: 10.1007/s10763-016-9764-4.
* Escudero, I., & Sánchez, V. (2007). How do domains of knowledge integrate into mathematics teachers' practice? *The Journal of Mathematical Behavior, 26*(4), 312-327. http://dx.doi.org.ucd.idm.oclc.org/10.1016/j.jmathb.2007.11.002
* Essien, A. A. (2010). What teacher educators consider as best practices in preparing pre-service teachers for teaching mathematics in multilingual classrooms. *Perspectives in Education, 28*(4), 32-42.
* Feiman-Nemser, S. (2008). Teacher learning: how do teachers learn to teach? In M. Cochran-Smith, S., Feiman-Nemser, D. J. McIntyre, & K. E. Demers (Eds.), *Handbook of research on teacher education: Enduring questions in changing contexts* (3rd ed., pp. 697-705). New York: Routledge.
* Firmender, J. M., Gavin, M. K., & McCoach, D. B. (2014). Examining the relationship between teachers' instructional practices and students' mathematics achievement. *Journal of Advanced Academics, 25*(3), 214-236.
* Forzani, F. (2014). Understanding "core practices" and "practice-based" teacher education: Learning from the past. *Journal of Teacher Education, 65*(4) 357–368. doi: 10.1177/0022487114533800
* Franke, M. L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory into Practice*, *40*(2), 102-109. doi: 10.1207/s15430421tip4002_4.
* Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). USA: National Council of Teachers of Mathematics-Information Age Publishing Inc.
* Fyfe, E. R., & Rittle-Johnson, B. (2017). Mathematics practice without feedback: A desirable difficulty in a classroom setting. *Instructional Science, 45*(2), 177-194. http://dx.doi.org.ucd.idm.oclc.org/10.1007/s11251-016-9401-1.
* Gainsburg, J. (2012). Why new mathematics teachers do or don't use practices emphasized in their credential program. *Journal of Mathematics Teacher Education, 15*(5), 359-379. http://dx.doi.org.ucd.idm.oclc.org/10.1007/s10857-012-9208-1
* Gelzheiser, L. M., Griesemer, B. A., Pruzek, R. M., & Meyers, J. (2000). How are developmentally appropriate or traditional teaching practices related to the

mathematics achievement of general and special education students? *Early Education and Development, 11*(2), 217-238. http://dx.doi.org.ucd.idm.oclc.org/10.1207/s15566935eed1102_6
Gaudin, C., & Chaliès, S. (2015). Video viewing in teacher education and professional development: A literature review. *Educational Research Review, 16,* 41-67. http://dx.doi.org/10.1016/j.edurev.2015.06.001
* Ghousseini, H. (2015). Core practices and problems of practice in learning to lead classroom discussions. *The Elementary School Journal*, *115*(3), 334-357.
* Ghousseini, H. (2017). Rehearsals of teaching and opportunities to learn mathematical knowledge for teaching. *Cognition and Instruction*, *35*(3), 188-211. doi: 10.1080/07370008.2017.1323903.
* Ghousseini, H., & Herbst, P. (2016). Pedagogies of practice and opportunities to learn about classroom mathematics discussions. *Journal of Mathematics Teacher Education*, *19*(1), 79-103. doi: 10.1007/s10857-014-9296-1.
Gitomer, D. H., & Zisk, R. C. (2015). Knowing what teachers know. *Review of Research in Education, 39*(1), 1-53. doi: 10.3102/0091732X14557001
* Goodchild, S., Fuglestad, A. B., & Jaworski, B. (2013). Critical alignment in inquiry-based practice in developing mathematics teaching. *Educational Studies in Mathematics, 84*(3), 393-412.
Goodwin, C. (1994). Professional vision. *American Anthropologist*, *96*(3), 606-633. http://dx.doi.org/10.1525/aa.1994.96.3.02a00100
* Griffin, C. C., League, M. B., Griffin, V. L., & Bae, J. (2013). Discourse practices in inclusive elementary mathematics classrooms. *Learning Disability Quarterly, 36*(1), 9-20. http://dx.doi.org.ucd.idm.oclc.org/10.1177/0731948712465188
* Grossman, P. (ED.) (2018). *Teaching core practices in teacher education.* Cambridge, MA: Harvard Education Press.
* Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record, 111*(9), 2055–2100.
Grossman, P., Hammerness, K., & McDonald, M., (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching: Theory and Practice*, 15(2), 273-289.
* Grossman, P., Kavanagh, S. S., Dean, C. G. P. (2018). The turn towards practice-based education: Introduction to the work of the Core Practices Consortium. In P. Grossman (Ed.), *Teaching core practices in teacher education* (pp. 1-14). Cambridge, MA: Harvard Education Press.
Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal, 45*(1), 184-205.
* Gujarati, J. (2013). An "inverse" relationship between mathematics identities and classroom practices among early career elementary teachers: The impact of accountability. *The Journal of Mathematical Behavior, 32*(3), 633-648. http://dx.doi.org.ucd.idm.oclc.org/10.1016/j.jmathb.2013.08.002
* Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Toward a new equity research agenda. *Mathematical Thinking and Learning, 4*(2-3), 145-187. http://dx.doi.org.ucd.idm.oclc.org/10.1207/S15327833MTL04023_4
* Han, X., & Paine, L. (2010). Teaching mathematics as deliberate practice through public lessons. *The Elementary School Journal, 110*(4), 519-541. http://dx.doi.org.ucd.idm.oclc.org/10.1086/651194

Hatch, T. & Grossman P. (2009). Learning to look beyond the boundaries of representation: Using technology to examine teaching (Overview for a digital exhibition: Learning from the practice of teaching). *Journal of Teacher Education* 60(1), 70-85.
* Heller, V. (2015). Academic discourse practices in action: Invoking discursive norms in mathematics and language lessons. *Linguistics and Education, 31*, 187-206. http://dx.doi.org.ucd.idm.oclc.org/10.1016/j.linged.2014.12.003
* Herbel-Eisenmann, B., Lubienski, S. T., & Id-Deen, L. (2006). Reconsidering the study of mathematics instructional practices: The importance of curricular context in understanding local and global teacher change. *Journal of Mathematics Teacher Education, 9*(4), 313-345. http://dx.doi.org.ucd.idm.oclc.org/10.1007/s10857-006-9012-x
* Herbst, P., Chieu, V., & Rougée, A. (2014). Approximating the practice of mathematics teaching: What learning can web-based, multimedia storyboarding software enable? *Contemporary Issues in Technology and Teacher Education (CITE Journal), 14*(4), 356-383.
* Herbst, P., Chazan, D., Kosko, K. W., Dimmel, J., & Erickson, A. (2016). Using multimedia questionnaires to study influences on the decisions mathematics teachers make in instructional situations. *ZDM Mathematics Education*, 48(1-2), 167-183. doi: 10.1007/s11858-015-0727-y.
* Herbst, P., & Kosko, K. W. (2014). Using representations of practice to elicit mathematics teachers' tacit knowledge of practice: A comparison of responses to animations and videos. *Journal of Mathematics Teacher Education*, *17*(6), 515-537. doi: 10.1007/s10857-013-9267-y.
* Hertzog, H. S., & O'Rode, N. (2011). Improving the quality of elementary mathematics student teaching: Using field support materials to develop reflective practice in student teachers. *Teacher Education Quarterly, 38*(3), 89-111.
* Hill, H. C. (2004). Professional development standards and practices in elementary school mathematics. *The Elementary School Journal, 104*(3), 215-231. http://dx.doi.org.ucd.idm.oclc.org/10.1086/499750
* Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430-511, doi: 10.1080/07370000802177235.
* Hlas, A. C., & Hlas, C. S. (2012). A review of high-leverage teaching practices: Making connections between mathematics and foreign languages. *Foreign Language Annals*, *45*(S1), 76-97. doi: 10.111/j.1944-9720.2012.01180.x.
* Hong, H., & Chai, C. S. (2017). Principle-based design: Development of adaptive mathematics teaching practices and beliefs in a knowledge building environment. *Computers & Education, 115*, 38-55. http://dx.doi.org.ucd.idm.oclc.org/10.1016/j.compedu.2017.07.011
Horizon Research (2000). *Inside the classroom observation and analytic protocol*. Available at: http://www.horizon-research.com/instruments/clas/cop.pdf. Accessed 1 December 2018.
* Horn, I. S. (2010). Teaching replays, teaching rehearsals, and re-visions of practice: Learning from colleagues in a mathematics teacher community. *Teachers College Record*, *112*(1), 225-259.

* House, J. D. (2002). Instructional practices and mathematics achievement of adolescent students in Chinese Taipei: Results from the TIMSS 1999 assessment. *Child Study Journal, 32*(3), 157-178.
* Huang, R., & Shimizu, Y. (2016). Improving teaching, developing teachers and teacher educators, and linking theory and practice through lesson study in mathematics: An international perspective. *ZDM Mathematics Education*, *48*(4), 393-409. doi: 10.1007/s11858-016-0795-7.
* Huang, R., Barlow, A. T., & Haupt, M. E. (2017). Improving core instructional practice in mathematics teaching through lesson study. *International Journal for Lesson and Learning Studies*, *6*(4), 365-379. doi: 10.1108/IJLLS-12-2016-0055.
* Hughes, E. M., Powell, S. R., Lembke, E. S., & Riley-Tillman, T. C. (2016). Taking the guesswork out of locating evidence-based mathematics practices for diverse learners. *Learning Disabilities Research & Practice, 31*(3), 130-141. http://dx.doi.org.ucd.idm.oclc.org/10.1111/ldrp.12103
* Ing, M., Webb, N. M., Franke, M. L., Turrou, A. C., Wong, J., Shin, N., & Fernandez, C. H. (2015). Student participation in elementary mathematics classrooms: The missing link between teacher practices and student achievement? *Educational Studies in Mathematics*, *90*(3), 341-356. doi: 10.1007/s10649-015-9625-z.
* Jackson, K., & Wilson, J. (2012). Supporting African American students' learning of mathematics: A problem of practice. *Urban Education, 47*(2), 354-398. http://dx.doi.org.ucd.idm.oclc.org/10.1177/0042085911429083
* Jacobbe, T., Ross, D. D., Caron, A. D., Barko, T., & Busi, R. (2014). Connecting theory and practice: Preservice teachers' construction of practical tools for teaching mathematics. *Teacher Education and Practice, 27*(2), 24.
* Jacobs, V. R., & Spangler, D. A, (2017). Research on core practices in K-12 mathematics teaching. In J. Cai (Ed.), *First compendium for research in mathematics education* (pp. 766-792). Reston, VA: National Council of Teachers of Mathematics.
* Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9*(2), 187-211.
* Jao, L., Wiseman, D., Kobiela, M., Gonsalves, A., & Savard, A. (2018). Practice-based pedagogy in mathematics and science teaching methods: Challenges and adaptations in context. *Canadian Journal of Science, Mathematics and Technology Education,* 18(2), 177-186.
* Jong, C., Pedulla, J. J., Reagan, E. M., Salomon-Fernandez, Y., & Cochran-Smith, M. (2010). Exploring the link between reformed teaching practices and pupil learning in elementary school mathematics. *School Science and Mathematics, 110*(6), 309-326.
* Kaminski, E. (2003). Promoting pre-service teacher education students' reflective practice in mathematics. *Asia-Pacific Journal of Teacher Education, 31*(1), 21-32. http://dx.doi.org.ucd.idm.oclc.org/10.1080/13598660301619
* Karsenty, R., & Arcavi, A. (2017). Mathematics, lenses and videotapes: A framework and a language for developing reflective practices of teaching. *Journal of Mathematics Teacher Education*, *20*(5), 433-455. doi: 10.1007/s10857-017-9379-x.
* Kazemi, E., Ghousseini, H., Cunard, A., & Turrou, A. C. (2016). Getting inside rehearsals: Insights from teacher educators to support work on complex practice. *Journal of Teacher Education*, *67*(1), 18-31. doi: 10.1177/0022487115615191.
* Kelley-Petersen, M., Davis, E. A., Ghousseini, H., Kloser, M., & Monte-Sano C. (2018). Rehearsals as examples of approximation. In P. Grossman (Ed.), *Teaching core*

practices in teacher education (pp. 85-105). Cambridge, MA: Harvard Education Press.
* Kiemer, K., Gröschner, A., Pehmer, A., & Seidel, T. (2015). Effects of a classroom discourse intervention on teachers' practice and students' motivation to learn mathematics and science. *Learning and Instruction, 35*, 94-103. http://dx.doi.org.ucd.idm.oclc.org/10.1016/j.learninstruc.2014.10.003
* Kline, S. L., & Ishii, D. K. (2008). Procedural explanations in mathematics writing: A framework for understanding college students' effective communication practices. *Written Communication, 25*(4), 441-461. http://dx.doi.org.ucd.idm.oclc.org/10.1177/0741088308322343
* Kosko, K. W., & Wilkins, J. L. M. (2015). Does time matter in improving mathematical discussions? The influence of mathematical autonomy. *The Journal of Experimental Education*, *83*(3), 368-385. doi: 10.1080/00220973.2014.907225.
Lacelle-Peterson, M. W. & Rivera, C. (1994). Is it real for all kids? A framework for equitable assessment policies for English language learners. *Harvard Educational Review, 64*(1), 55-75.
Ladson-Billings, G. (1995). But that's just good teaching! The case for culturally relevant pedagogy. *Theory Into Practice 34*(3), 159-65.
* Lai, M. Y., Auhl, G., & Hastings, W. (2015). Improving pre-service teachers' understanding of complexity of mathematics instructional practice through deliberate practice: A case study on "study of teaching". *International Journal for Mathematics Teaching and Learning,* 1-25.
* Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven and London: Yale University.
* Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education*, *61*(1-2), 21-34. doi: 10.1177/0022487109347321.
Lampert, M., & Ball, D. L. (*1998*). *Teaching, multimedia, and mathematics: Investigations of real practice.* New York: Teachers College Press.
* Lampert, M., Boerst, T. A., & Graziani, F. (2011). Organizational resources in the service of school-wide ambitious teaching practice. *Teachers College Record*, *113*(7), 1361-1400.
* Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., ... & Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education*, *64*(3), 226-243. doi: 10.1177/0022487112 473837.
* Lan, X., Ponitz, C. C., Miller, K. F., Li, S., Cortina, K., Perry, M., & Fang, G. (2009). Keeping their attention: Classroom practices associated with behavioral engagement in first grade mathematics classes in China and the United States. *Early Childhood Research Quarterly, 24*(2), 198-211. http://dx.doi.org.ucd.idm.oclc.org/10.1016/j.ecresq.2009.03.002
* Lee, C. W., Walkowiak, T. A., & Nietfeld, J. L. (2017). Characterization of mathematics instructional practises for prospective elementary teachers with varying levels of self-efficacy in classroom management and mathematics teaching. *Mathematics Education Research Journal*, *29*(1), 45-72. doi: 10.1007/s13394-016-0185-z.
* Lerkkanen, M. K., Kiuru, N., Pakarinen, E., Viljaranta, J., Poikkeus, A. M., Rasku-Puttonen, H., ... & Nurmi, J. E. (2012). The role of teaching practices in the development of children's interest in reading and mathematics in

kindergarten. *Contemporary Educational Psychology*, 37(4), 266-279. doi: 10.1016/j.cedpsych.2011.03.004.
* Lerman, S., & Zehetmeier, S. (2008). Face-to-face communities and networks of practising mathematics teachers: Studies on their professional growth. In K. Krainer & T. Wood (Eds.), *The international handbook of mathematics teacher education: Participants in mathematics teacher education* (vol. 3, pp. 33-154). Rotterdam: Sense Publishers.
* Liang, S., Glaz, S., DeFranco, T., Vinsonhaler, C., Grenier, R., & Cardetti, F. (2013). An examination of the preparation and practice of grades 7–12 mathematics teachers from the Shandong province in China. *Journal of Mathematics Teacher Education, 16*(2), 149-160. http://dx.doi.org.ucd.idm.oclc.org/10.1007/s10857-012-9228-x
* Lloyd, G. M. (2014). Research into teachers' knowledge and the development of mathematics classroom practices. *Journal of Mathematics Teacher Education, 17*(5), 393-395. http://dx.doi.org.ucd.idm.oclc.org/10.1007/s10857-014-9285-4
* Lloyd, M. E. R. (2013). Transfer of practices and conceptions of teaching and learning mathematics. *Action in Teacher Education, 35*(2), 103-124.
Lortie, D. C. (1975). *Schoolteacher*. Chicago and London: The University of Chicago Press.
* Maccini, P., & Gagnon, J. C. (2006). Mathematics instructional practices and assessment accommodations by secondary special and general educators. *Exceptional Children*, *72*(2), 217-234.
* Maher, C. A. (2008). Video recordings as pedagogical tools in mathematics teacher education. In D. Tirosh & T. Wood (Eds.), *The international handbook of mathematics teacher education: Tools and processes in mathematics teacher education* (vol. 2, pp. 65-84). Rotterdam: Sense Publishers.
* Martínez, J. F., Stecher, B., & Borko, H. (2009). Classroom assessment practices, teacher judgments, and student achievement in mathematics: Evidence from the ECLS. *Educational Assessment, 14*(2), 78-102. http://dx.doi.org.ucd.idm.oclc.org/10.1080/10627190903039429
* Maryono, Sutawidjaja, A., & Irawati, S. (2017). Implementation of pedagogical content knowledge (PCK) of mathematics teachers in teaching practice: A case study. *International Education Studies, 10*(3), 11-25.
* Mayrowetz, D. (2009). Instructional practice in the context of converging policies: Teaching mathematics in inclusive elementary classrooms in the standards reform era. *Educational Policy, 23*(4), 554-588.
* McCaffrey, D. F., Hamilton, L. S., Stecher, B. M., Klein, S. P., Bugliari, D., & Robyn, A. (2001). Interactions among instructional practices, curriculum, and student achievement: The case of standards-based high school mathematics. *Journal for Research in Mathematics Education, 32*(5), 493-517. http://dx.doi.org.ucd.idm.oclc.org/10.2307/749803
* McClintock, E., O'Brien, G., & Jiang, Z. (2005). Assessing teaching practices of secondary mathematics student teachers: An exploratory cross case analysis of voluntary field experiences. *Teacher Education Quarterly, 32*(3), 139-151.
* McDonald, M., Kazemi, E., & Kavanagh, S. S. (2013). Core practices and pedagogies of teacher education: A call for a common language and collective activity. *Journal of Teacher Education*, *64*(5), 378-386. doi: 10.1177/0022487113493807.
* McDuffie, A. M. R., & Mather, M. (2006). Reification of instructional materials as part of the process of developing problem-based practices in mathematics education. *Teachers and Teaching: Theory and Practice, 12*(4), 435-459. http://dx.doi.org.ucd.idm.oclc.org/10.1080/13450600600644285

* McDuffie, A. R. (2004). Mathematics teaching as a deliberate practice: An investigation of elementary pre-service teachers' reflective thinking during student teaching. *Journal of Mathematics Teacher Education, 7*(1), 33-61. http://dx.doi.org.ucd.idm.oclc.org/10.1023/B:JMTE.0000009970.12529.f4
* McGrew, S., Alston, C. L., & Fogo, B. (2018). Modeling as an example of representation. In P. Grossman (Ed.), *Teaching core practices in teacher education* (pp. 35-55). Cambridge, MA: Harvard Education Press.
* McKinney, S. E., Robinson, J., & Berube, C. T. (2013). "Real teaching" in the mathematics classroom: A comparison of the instructional practices of elementary teachers in urban high-poverty schools. *Teacher Education and Practice, 26*(4), 797-815.
* Merritt, E. G., Palacios, N., Banse, H., Rimm-Kaufman, S., & Leis, M. (2017). Teaching practices in Grade 5 mathematics classrooms with high-achieving English learner students. *The Journal of Educational Research, 110*(1), 17-31. http://dx.doi.org.ucd.idm.oclc.org/10.1080/00220671.2015.1034352
* Miller, S. P., & Hudson, P. J. (2007). Using evidence-based practices to build mathematics competence related to conceptual, procedural, and declarative knowledge. *Learning Disabilities Research & Practice, 22*(1), 47-57. http://dx.doi.org.ucd.idm.oclc.org/10.1111/j.1540-5826.2007.00230.x
* Morgan, P. L., Farkas, G., & Maczuga, S. (2015). Which instructional practices most help first-grade students with and without mathematics difficulties? *Educational Evaluation and Policy Analysis, 37*(2), 184-205. http://dx.doi.org.ucd.idm.oclc.org/10.3102/0162373714536608
* Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education, 6*(1), 45-57.
* Ottmar, E. R., Rimm-Kaufman, S. E., Berry, R. Q., & Larsen, R. A. (2013). Does the responsive classroom approach affect the use of standards-based mathematics teaching practices? *The Elementary School Journal*, *113*(3), 434-457.
* Ottmar, E. R., Rimm-Kaufman, S., Larsen, R. A., & Berry, R. Q. (2015). Mathematical knowledge for teaching, standards-based mathematics teaching practices, and student achievement in the context of the responsive classroom approach. *American Educational Research Journal, 52*(4), 787-821. http://dx.doi.org.ucd.idm.oclc.org/10.3102/0002831215579484
Peercy, M. M., & Troyan, F. J. (2107). Making transparent the challenges of developing a practice-based pedagogy of teacher education. *Teaching and Teacher Education, 61*, 26-36. http://dx.doi.org/10.1016/j.tate.2016.10.005
Praetorius, A.K., & Charalambous, C. Y. (2018). Classroom observation frameworks for studying instructional quality: looking back and looking forward. *ZDM Mathematics Education, 50*(3), 535-553. DOI 10.1007/s11858-018-0946-0
Reynolds, D., Chapman, C., Clarke, P., Muijs, D., Sammons, P., & Teddlie, C. (2016). Conclusions: The future of educational effectiveness and improvement research, and some suggestions and speculations. In C. Chapman, D. Muijs, D. Reynolds, P. Sammons, and C. Teddlie (Eds.), *The Routledge international handbook of educational effectiveness and improvement: Research, policy, and practice* (pp. 408-439). New York, NY: Routledge.
* Rosenquist, B. A., Henrick, E. C., & Smith, T. M. (2015). Research–practice partnerships to support the development of high quality mathematics instruction for all

students. *Journal of Education for Students Placed at Risk, 20*(1-2), 42-57. http://dx.doi.org.ucd.idm.oclc.org/10.1080/10824669.2014.988335
* Russ, R. S., Sherin, B. L., & Sherin, M. G. (2016). What constitutes teacher learning? In D. H. Gitomer, & C. A. Bell (Eds.), *Handbook of research on teaching* (5th ed., pp. 391-438). Washington, DC: American Educational Research Association.
* Santagata, R. (2005). Practices and beliefs in mistake-handling activities: A video study of Italian and US mathematics lessons. *Teaching and Teacher Education, 21*(5), 491-508. http://dx.doi.org.ucd.idm.oclc.org/10.1016/j.tate.2005.03.004
* Santagata, R., & Yeh, C. (2014). Learning to teach mathematics and to analyze teaching effectiveness: Evidence from a video- and practice-based approach. *Journal of Mathematics Teacher Education, 17*(6), 491-514. http://dx.doi.org.ucd.idm.oclc.org/10.1007/s10857-013-9263-2
* Schutz, K. M., Grossman, P., & Shaughnessy, M. (2018). Approximations of practice in teacher education. In P. Grossman (Ed.), *Teaching core practices in teacher education* (pp. 57-83). Cambridge, MA: Harvard Education Press.
* Shaughnessy, M., & Boerst, T. A. (2018). Uncovering the skills that preservice teachers bring to teacher education: The practice of eliciting a student's thinking. *Journal of Teacher Education*, *69*(1), 40-55. doi: 10.1177/0022487117702574.
* Shechtman, N., Roschelle, J., Haertel, G., & Knudsen, J. (2010). Investigating links from teacher knowledge, to classroom practice, to student learning in the instructional system of the middle-school mathematics classroom. *Cognition and Instruction, 28*(3), 317-359. ttp://dx.doi.org.ucd.idm.oclc.org/10.1080/07370008.2010.487961
* Sileo, J. M., & van Garderen, D. (2010). Creating optimal opportunities to learn mathematics: Blending co-teaching structures with research-based practices. *Teaching Exceptional Children, 42*(3), 14-21.
* Silver, E. (2010). Examining what teachers do when they display their best practice: Teaching mathematics for understanding. *Journal of Mathematics Education at Teachers College, 1*(1), 1-6.
* Silver, E. A., Clark, L. M., Ghousseini, H. N., Charalambous, C. Y., & Sealy, J. T. (2007). Where is the mathematics? Examining teachers' mathematical learning opportunities in practice-based professional learning tasks. *Journal of Mathematics Teacher Education, 10*(4-6), 261-277. http://dx.doi.org.ucd.idm.oclc.org/10.1007/s10857-007-9039-7
* Sleep, L. (2012). The work of steering instruction toward the mathematical point: A decomposition of teaching practice. *American Educational Research Journal*, *49*(5), 935-970. doi: 10.3102/0002831212448095.
* Sleep, L., & Boerst, T. A. (2012). Preparing beginning teachers to elicit and interpret students' mathematical thinking. *Teaching and Teacher Education*, *28*(7), 1038-1048. doi: 10.1016/j.tate.2012.04.005.
Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* Reston, VA: National Council of Teachers of Mathematics.
* Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157-223). USA: National Council of Teachers of Mathematics- Information Age Publishing Inc.
* Spooner, F., Saunders, A., Root, J., & Brosh, C. (2017). Promoting access to common core mathematics for students with severe disabilities through mathematical problem solving. *Research and Practice for Persons with Severe Disabilities*, *42*(3), 171-186. doi: 10.1177/1540796917697119.

* Staples, M. E., Bartlo, J., & Thanheiser, E. (2012). Justification as a teaching and learning practice: Its (potential) multifacted role in middle grades mathematics classrooms. *The Journal of Mathematical Behavior, 31*(4), 447-462. http://dx.doi.org.ucd.idm.oclc.org/10.1016/j.jmathb.2012.07.001
Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*, 310-340.
Stigler, J.W. & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers' for improving education in the classroom.* New York: The Free Press.
Stipek, D. & Byler, P. (2004). The early childhood classroom observation measure. *Early Childhood Research Quarterly*, 19, 375 – 397.
* Stockero, S. L., & Van Zoest, L. R. (2013). Characterizing pivotal teaching moments in beginning mathematics teachers' practice. *Journal of Mathematics Teacher Education, 16*(2), 125-147. http://dx.doi.org.ucd.idm.oclc.org/10.1007/s10857-012-9222-3
Stylianides, A. J. & Delaney, S. (2011). The cultural dimension of teachers' mathematical knowledge. In T. Rowland & K. Ruthven, K. (Eds.), *Mathematical knowledge in teaching* (Mathematics Education Library, vol. 50, pp. 179-191). Springer, Dordrecht.
* Suurtamm, C., Koch, M., & Arden, A. (2010). Teachers' assessment practices in mathematics: Classrooms in the context of reform. *Assessment in Education: Principles, Policy & Practice, 17*(4), 399-417. http://dx.doi.org.ucd.idm.oclc.org/10.1080/0969594X.2010.497469
* Swars, S. L., Smith, S. Z., Smith, M. E., Carothers, J., & Myers, K. (2018). The preparation experiences of elementary mathematics specialists: Examining influences on beliefs, content knowledge, and teaching practices. *Journal of Mathematics Teacher Education*, *21*(2), 123-145. doi: 10.1007/s10857-016-9354-y.
Tarr, J. E., Reys, R.E., Reys, B. J., Chávez, Ó., Shih, J., & Osterlind, S.J. (2008). The impact of middle grade mathematics curricula and the classroom learning environment on student achievement. *Journal for Research in Mathematics Education*, *39*(3), 247 – 280.
* Tchoshanov, M. A. (2011). Relationship between teacher knowledge of concepts and connections, teaching practice, and student achievement in middle grades mathematics. *Educational Studies in Mathematics, 76*(2), 141-164.
* Thomas, K. (2013). Changing mathematics teaching practices and improving student outcomes through collaborative evaluation. *Teacher Education and Practice, 26*(4), 779-796.
* Tyminski, A. M., Zambak, V. S., Drake, C., & Land, T. J. (2014). Using representations, decomposition, and approximations of practices to support prospective elementary mathematics teachers' practice of organizing discussions. *Journal of Mathematics Teacher Education*, *17*(5), 463-487. doi: 10.1007/s10857-013-9261-4.
* Ukpokodu, O. N. (2011). How do I teach mathematics in a culturally responsive way?: Identifying empowering teaching practices. *Multicultural Education, 19*(3), 47-56.
* Wager, A. A. (2012). Incorporating out-of-school mathematics: From cultural context to embedded practice. *Journal of Mathematics Teacher Education, 15*(1), 9-23. http://dx.doi.org.ucd.idm.oclc.org/10.1007/s10857-011-9199-3
* Webb, N. M., Franke, M. L., De, T., Chan, A. G., Freund, D., Shein, P., & Melkonian, D. K. (2009). 'Explain to your partner': Teachers' instructional practices and students'

dialogue in small groups. *Cambridge Journal of Education*, *39*(1), 49-70. doi: 10.1080/03057640802701986.
* Webb, N. M., Franke, M. L., Ing, M., Turrou, A. C., Johnson, N. C., & Zimmerman, J. (2017). Teacher practices that promote productive dialogue and learning in mathematics classrooms. *International Journal of Educational Research*, 1-11. doi: 10.1016/j.ijer.2017.07.009.
* Webb, N. M., Franke, M. L., Ing, M., Wong, J., Fernandez, C. H., Shin, N., & Turrou, A. C. (2014). Engaging with others' mathematical ideas: Interrelationships among student participation, teachers' instructional practices, and learning. *International Journal of Educational Research*, *63*, 79-93. doi: 10.1016/j.ijer.2013.02.001.
* Wieman, C., & Gilbert, S. (2014). The teaching practices inventory: A new tool for characterizing college and university teaching in mathematics and science. *CBE-Life Sciences Education, 13*(3), 552-569. doi: 10.1187/cbe.14-02-0023.
* Xenofontos, C. (2016). Teaching mathematics in culturally and linguistically diverse classrooms: Greek-Cypriot elementary teachers' reported practices and professional needs. *Journal of Urban Mathematics Education, 9*(1), 94-116.
* Yeh, C. (2017). Math is more than numbers: Beginning bilingual teachers' mathematics teaching practices and their opportunities to learn. *Journal of Urban Mathematics Education, 10*(2), 106-139.
* Yu, R., & Singh, K. (2018). Teacher support, instructional practices, student motivation, and mathematics achievement in high school. *The Journal of Educational Research*, *111*(1), 81-94. doi: 10.1080/00220671.2016.1204260.
* Zahner, W., Velazquez, G., Moschkovich, J., Vahey, P., & Lara-Meloy, T. (2012). Mathematics teaching practices with technology that support conceptual understanding for Latino/a students. *The Journal of Mathematical Behavior, 31*(4), 431-446. http://dx.doi.org.ucd.idm.oclc.org/10.1016/j.jmathb.2012.06.002
* Zeichner, K. (2012). The turn once again toward practice-based teacher education. *Journal of Teacher Education, 63*(5), 376-382.