Pre-service mathematics teachers' knowledge and beliefs

Andreas J. Stylianides, University of Cambridge, UK
Seán Delaney, Marino Institute of Education, Ireland

A large body of research in mathematics education and mathematics teacher education has examined different issues related to mathematics teachers' knowledge and beliefs, how they are acquired, how they are changed, and how they affect student learning. The research focuses in some cases on one of these constructs – either knowledge (e.g., Ball, Thames, & Phelps, 2008) or beliefs (e.g., Philipp, 2007) – and in other cases on the interplay between the two (e.g., Drageset, 2010). As we will illustrate shortly, there is no canonical definition of mathematics teachers' knowledge or beliefs in the relevant literature. However, a common thread that permeates this literature is that mathematics teachers' knowledge and beliefs (however construed) are associated with teachers' instructional decisions and thus influence students' opportunities to learn mathematics. According to Wilson and Cooney (2002),

> regardless of whether one calls teacher thinking beliefs, knowledge, conceptions, cognitions, views, or orientations, with all the subtlety these terms imply, or how they are assessed, e.g., by questionnaires (or other written means), interviews, or observations, the evidence is clear that teacher thinking influences what happens in classrooms, what teachers communicate to students, and what students ultimately learn (p. 144).

The various conceptualizations of teachers' mathematical knowledge and beliefs that are available in the literature reflect different perspectives researchers have brought to the study of these constructs. Ponte and Chapman (2008) conceptualized *knowledge* broadly "to refer to a wide network of concepts, images, and intelligent abilities possessed by human beings, including beliefs and conceptions" (p. 233), and they distinguished between two main and partly overlapping kinds of teacher knowledge: *knowledge of mathematics*, which has a referent in the field of mathematics, and *knowledge of mathematics teaching*, which has a referent in professional practice. The perspective of Ponte and Chapman (2008) to consider *beliefs* as an element of the broader construct of *knowledge* is reflected also in the writings of other researchers. For example, Philipp (2007) viewed knowledge as comprising the special class of beliefs that are "held with certainty" (p. 259). Similarly, Furinghetti and her colleagues associated beliefs with individuals' subjective (personal) knowledge in contrast with the kind of objective (official) knowledge accepted within a community (Furinghetti & Pehkonen, 2002), or as a main component of a teacher's "practical knowledge" (Furinghetti & Morselli, 2011).

Other researchers considered separately the constructs of teachers' knowledge and teachers' beliefs and offered categorizations of each thus illuminating different (often complementary) aspects of their complex and multifaceted natures. Regarding teachers' knowledge, following Shulman's (1986) influential work, several frameworks have been developed to describe important components of the content knowledge that teachers of mathematics draw on, or need to have, as they manage the demands of their professional

practice. Two examples of such frameworks are the Mathematical Knowledge for Teaching framework (Ball et al., 2008) and the Knowledge Quartet framework (Rowland, Turner, Thwaites, & Huckstep, 2009). These frameworks have supported several strands of research in the area of mathematics teachers' content knowledge, such as research on the relationship between teachers' content knowledge and students' achievement (Hill, Rowan, & Ball, 2005), on deepening teachers' content knowledge for teaching (Turner & Rowland, 2011), or on organizing the mathematical preparation of pre-service teachers in teacher education (Stylianides & Stylianides, 2014b).

Regarding teachers' beliefs, several frameworks have been developed to categorize those beliefs primarily according to their objects. Furinghetti and Morselli (2011) noted that "[t]he objects of mathematics teachers' beliefs may be internal (themselves as persons, as learners, as teachers) or external (the nature of mathematics, the nature of mathematics teaching and learning)" (p. 589). An example of a framework addressing internal-objects beliefs is Bandura's (1977, 1997) framework of self-efficacy beliefs as used in research on teachers' self-efficacy beliefs in specific mathematical domains such as problem solving (e.g., Stylianides & Stylianides, 2014a) or on teachers' self-efficacy beliefs about mathematics teaching more generally (e.g., Philippou & Christou, 1998). An example of a framework addressing external-object beliefs is Ernest's (1989) framework identifying different teacher roles in the classroom, such as facilitator or instructor. A major strand of research on mathematics teachers' beliefs has explored connections between teachers' beliefs and their teaching practice, though inconsistencies were often documented between the two thus motivating the development of further conceptualizations about the nature of teachers' beliefs, such as viewing teachers' beliefs as sensible systems (Leatham, 2006).

The bottom line of this brief overview of research on mathematics teachers' knowledge and beliefs is that, despite their manifold conceptualizations in the literature, the constructs of knowledge and beliefs are important factors to consider both in the study of classroom instruction in mathematics and in thinking about the goals, curriculum, or organization of the education of pre-service mathematics teachers. The four chapters in this section of the book add considerably to this body of research; they illustrate some of the avenues currently being pursued within it and identify some that merit further investigation, with a particular focus on pre-service primary teachers.

Specifically, the four chapters contribute, collectively, to the broad issue of describing, elaborating, or conceptualizing kinds of mathematical knowledge and beliefs that are important for the education of pre-service primary teachers. In doing so, they raise some interesting challenges for the curriculum of teacher education and research in this area. We organize our commentary around three sections according to whether the focus of the reviewed chapters is on teacher knowledge or teacher beliefs only, or on the interplay between the two.

We acknowledge that the focus of our commentary on teacher knowledge and beliefs inevitably downplays some important contributions made in the chapters that did not fit directly within the scope of our commentary. We will allude to some of these contributions as we discuss each chapter in the following sections or in the final section where we will consider implications of the four chapters for teacher education research and practice.

Two chapters focus primarily on teacher knowledge. Shinno et al. looked at how prospective teachers used their mathematical knowledge to evaluate student answers to a realistic mathematics problem. They used their findings to elaborate on the Ball et al (2008) model of mathematical knowledge for teaching by including components of mathematical argumentation as part of what teachers need to know. Çelik et al. looked at teacher knowledge from the perspective of teaching diverse students. They adapted the Teaching for Diversity scale from the instrument developed for the TEDS-M study and administered the instrument to prospective primary teachers in Turkey. They found that prospective teachers have insufficient opportunities to develop strategies and curriculum to support the learning of students from diverse cultural and intellectual backgrounds.

The chapter by Güneş focuses on teachers' beliefs. Specifically she studied the relationship between pre-service teachers' mathematical backgrounds and their perceptions of self-efficacy in mathematics. Overall the pre-service teachers perceived their self-efficacy in mathematics to be high and this was relatively stable across the four years of their initial teacher preparation program. Some relationship was found between the prospective teachers' past mathematical experiences and their self-efficacy perceptions.

In the fourth chapter Jacobsen et al. conceive of mathematical proficiency for teaching as including teacher knowledge and belief constructs related to students' mathematics learning. In order to investigate how the dynamic characteristics of knowledge and disposition change over time, they focused on the specific area of multidigit addition and subtraction. They found that pre-service teachers' knowledge and self-efficacy beliefs developed during a methods course but their motivation beliefs did not. They conjectured that components of mathematical proficiency for teaching may be topic-specific and that some components of proficiency may develop independently from other co-requisite or seemingly-related components.

In the remainder of this chapter we discuss these chapters both in relation to one another and in relation to wider research in the field. We identify implications of the work for teacher education and for ongoing research in this area.

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