Teaching Number in Senior Primary School Classes:

Insights from Research and Practice

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Teaser

Which of the following best predicts success in post-primary school mathematics?

(a) Knowledge of multiplication, addition and subtraction

(b) Working memory

(c) Family education

(d) Family income

(e) Knowledge of fractions and division of whole numbers

(f) Verbal and non-verbal IQ

Response to Teaser

Which of the following best predicts success in post-primary school mathematics?

(a) Knowledge of multiplication, addition and subtraction

(b) Working memory

(c) Family education

(d) Family income

(e) Knowledge of fractions and division of whole numbers (Siegler et al, 2012)

(f) Verbal and non-verbal IQ

My former approach to teaching maths

- Followed the textbook chapters in the sequence presented
- Tried to be clear when explaining new concepts (e.g. prime numbers or reciprocals) or procedures (e.g. long division, adding fractions)
- Highlighted key ideas on charts around the room (e.g. Rules for finding the Interest, principal, time or rate when teaching interest; rules for converting fractions, decimals and percentages; rule for expressing one quantity as a fraction of another; BOMDAS)
- Concrete materials and extra attention for low achievers and more word problems for high achievers.
- Regular revision (using mental maths books) and testing (mainly using home-made tests)
- Very similar to how I had been taught mathematics in primary and post-primary school
- But something was missing…

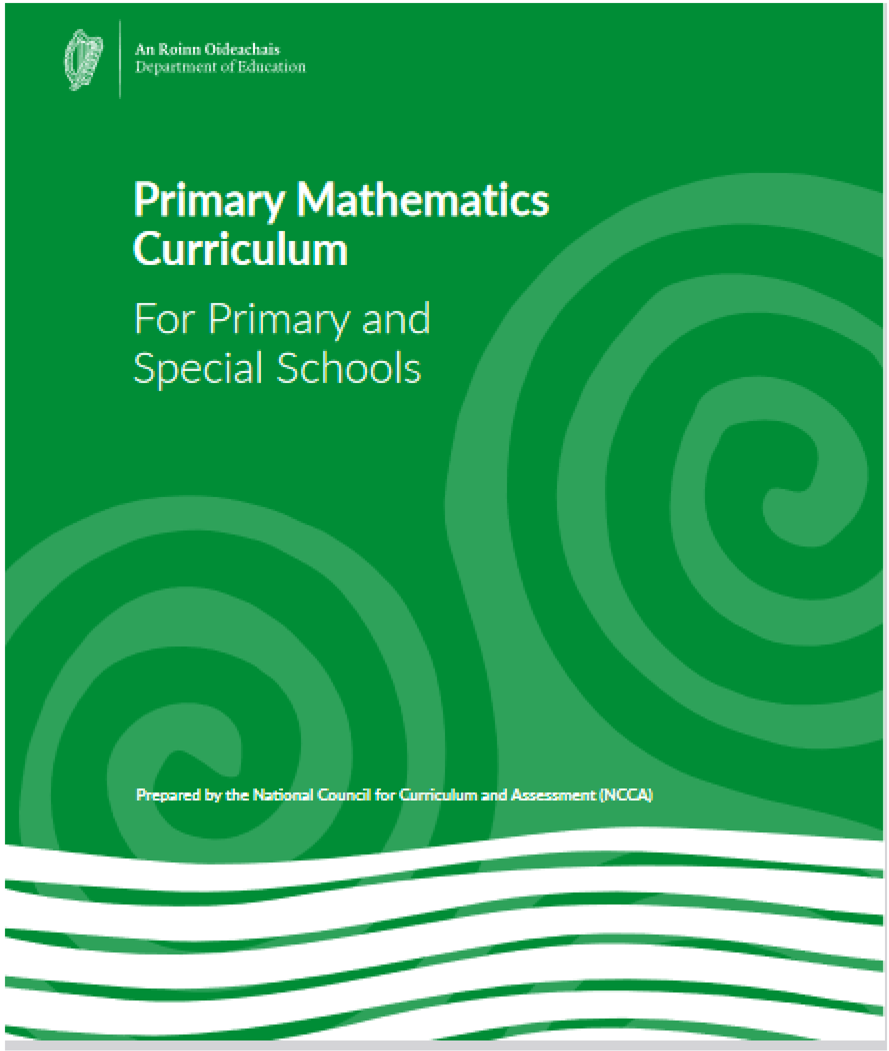
My Claim

If you implement the teaching approach presented here, you can help children in your class learn more mathematics and perform better in maths tests because you and the children will enjoy maths lessons more and the children will better connect their learning to what they already know.

New Mathematics Curriculum for Special and Primary Schools

Key pedagogical Practices

- Fostering a productive disposition
- Promoting maths talk
- Using cognitively challenging tasks
- Emphasising mathematical modelling
- Encouraging playfulness



Sense Making in Mathematics

- Teaching mathematics for understanding and learning mathematics with understanding (Hiebert et al, 1997; Silver et al, 2009).
- Widespread agreement that making sense of problems helps support students' fluency and flexibility in conceptually rich mathematics activities. (Buenrostro & Ehrenfeld, 2023).
- Some evidence that teaching for understanding promotes student achievement; extensive evidence for the strength of this effect is difficult to find (e.g. O'Dwyer et al, 2015).
- Absence of evidence is not evidence that such an approach does not work. Making sense is widely advocated by researchers but intermittently practised by teachers (e.g. Lubienski, 2011; O'Shea & Leavy, 2013; Slattery & Fitzmaurice, 2014).

Outline of Webinar

- Explore what sense making looks like in mathematics teaching
- Illustrations of sense making from two classrooms
- Ways in which sense making with numbers can be difficult in mathematics
- Making sense when teaching fractions
- Making sense when problem solving
- Possible objections to sense making in mathematics
- Q & A

What sense making in mathematics teaching is … and is not (Fitzgerald and Palincsar, 2019)

- Engage in a social process that is active, self-conscious, motivated, purposeful
- Adopt an exploratory stance towards the subject
- Construct meaning by applying important mathematical concepts, symbols and rules; explain, justify and evaluate solutions.
- Use reasoning
- Focus on process of mathematical sense making.

In contrast to

- Teaching to the test
- Learning off rules (Use the magic “0” in long multiplication; invert the divisor and multiply, etc.)
- Cramming for an exam
- Drill and practice
- Covering the textbook
- Learning disconnected facts

How teachers promote sense making in mathematics (Fitzgerald & Palincsar, 2019)

- Design tasks to promote opportunities for sense making.
- Make students' ideas public.
- Question students to extend and clarify their thinking; ask students to explain, reason and justify responses.
- Listen carefully to students
- Make connections between activities and concepts, and among ideas; develop and track students' prior and developing knowledge
- Raise challenge – requiring students to elaborate on answers, promoting high levels of reasoning, sustaining pressure on students to continue exploring a problem.
- Enculturate students into engaging in sense making conversations.
- Differentiate instruction – more time for some students; mixing whole-class and small-group instruction; increasing explicitness.

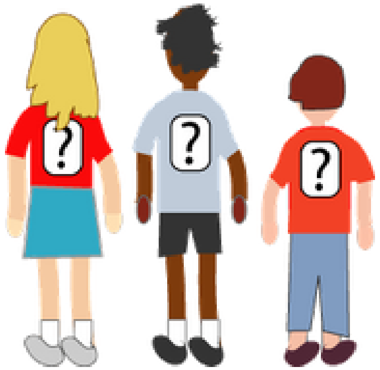
Teacher content knowledge of mathematics is important for promoting sense making.

Sample sense-making tasks 1

One Wasn't Square

Age 7 to 11
Challenge Level ★★

Mrs Morgan, the class's teacher, pinned numbers onto the backs of three children: Mona, Bob and Jamie.



"Now", she said, "Those three numbers add to a special kind of number. What is it?"

Michael put his hand up.

"It's a square number", he answered.

"Correct", smiled Mrs Morgan.

"Oh!" exclaimed Mona, "The two numbers I can see also add to a square!"

"And me!" called out Bob, "The two numbers I can see add to a square too!"

"Oh dear", said Jamie disappointedly, "the two numbers I can see don't add to a square! It's either **5** too little or **6** too big!"

What numbers did the three children have on their backs?

This one: https://nrich.maths.org/1119/index and more at https://nrich.maths.org/12635

Sample sense-making tasks 2

Task 10

Solve this problem with a partner.

Dublin Zoo has just received two new sheep for the Family Farm part of the zoo. The zoo keeper wants to build an enclosure for the sheep. She decides that the enclosure must be square or rectangular with an area of exactly 100 square metres.

(i) Which different enclosures could she build?

(ii) How many metres of fencing will she need for each possible enclosure?

(iii) Use your copy or some graph paper to draw all the possible rectangular or square enclosures.

(iv) Include a key to tell how much each unit on your copy or graph paper equals.

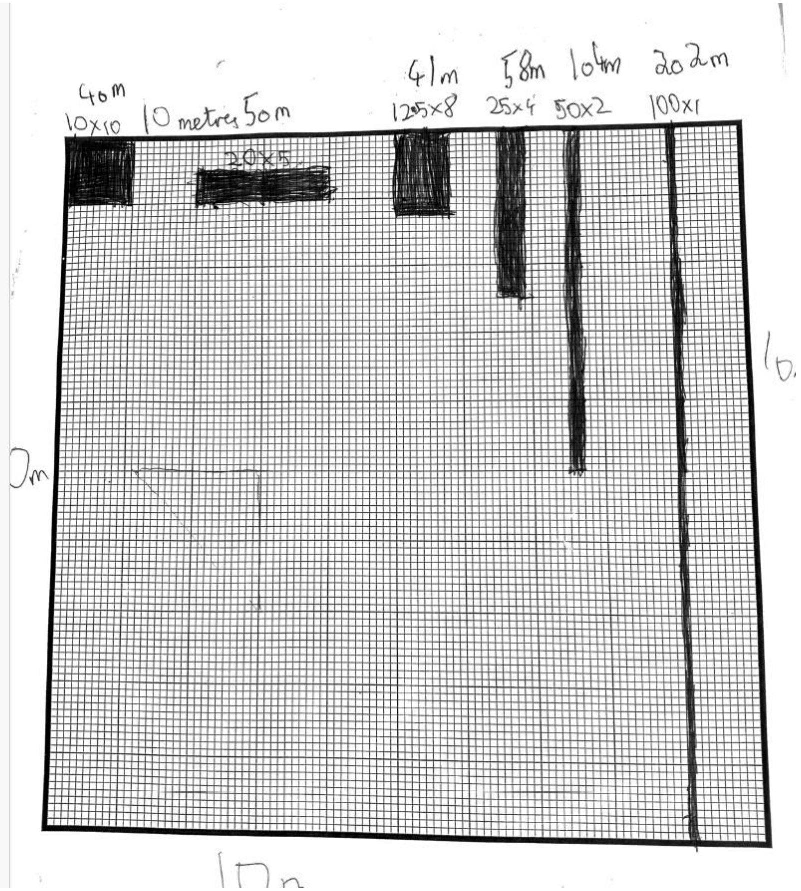
(v) Which enclosure would you recommend that the zoo keeper builds? Why?

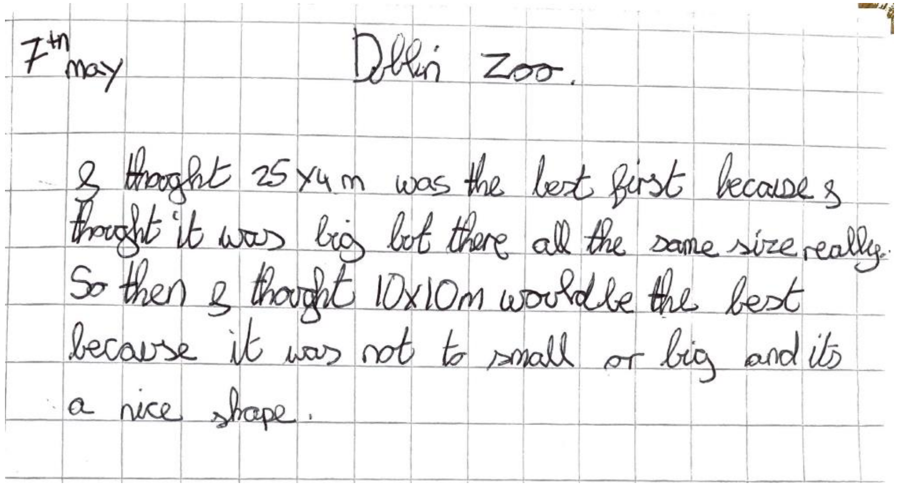
This and more from
http://seandelaney.com/wp-content/uploads/2013/11/PDSTArea5Maths-Pupils-1.pdf

Classroom Episode 1

5th Class

Recommending configurations for Family Farm in Dublin Zoo




7th may Dublin Zoo.

I thought 25x4m was the best first because I thought it was big but there all the same size really. So then I thought 10x10m would be the best because it was not to small or big and its a nice shape.

Classroom Episode 1

Julie:	Well, we think the twenty five metre and four metre across because it's a rectangle and we just think it's a good shape.
Karen:	And we also did fifty and two, two across and fifty down.
Teacher:	Okay, so you did one that was fifty metres long and two, two metres wide. This one here. Okay so it's fifty metres long and two metres wide, that's this one here, okay.

Teacher:	Daniel has a question for you.
Daniel:	Why do you do those two because the perimeter of those is quite bigger than a few of the other ones, and they could be more expensive to buy fencing for them?
Karen:	I don't understand what you're saying.
Teacher:	Can you explain yourself a little bit more clearly Daniel?
Daniel:	Alright, well the perimeter of those two are bigger
Teacher:	Of which two now?
Daniel:	Of the two type of fields that they're ...
Teacher:	That they're recommending
Daniel:	That they're recommending
Daniel:	Three of the other ones have smaller perimeters and it would be cheaper to buy fencing for those three apart from your two which are very, quite expensive.
Teacher:	Do you understand what Daniel is saying?
Julie:	Yeah.
Teacher:	So, what is he saying?
Julie:	The other three that we did that we didn't recommend why didn't we pick them?
Teacher:	Why does he think you might have chosen another one rather than the one you did?
Karen:	Because it would be more expensive
Teacher:	What would be more expensive?
Julie:	The fencing.
Teacher:	For which for which configuration?
Karen	For the ones that we did recommend.
Teacher:	Is that what you're saying?
Daniel:	Yeah, the ones that they were saying were...

Classroom Episode 1

Teacher:	**Let's just take one of them. I think the one you both agreed on was twenty five metres by four metres isn't that right? So, what's your point Daniel? Can I ask you all just to listen to this conversation? Maybe just leave down the water and things like that for the moment pens and pencils and just concentrate, 'cause I know it's important that you concentrate on this.**
Daniel:	That is the longer perimeter
Teacher:	Twenty five multiplied by four.
Daniel:	Yeah. Twenty five multiplied by four say, say that twelve point five multiplied by eight, twenty by five or ten by five and that means that it will be more expensive to buy fencing for that one than those three. So, why did you pick that one?
Karen:	Amm, I picked that one...
Teacher:	Sorry, can I ask Dylan can you actually just put into your own words what Daniel just said?
Dylan:	He said; why did you pick them two 'cause they would be a lot more expensive if you didn't pick them like if you picked the other one it would be a lot cheaper.
Teacher:	And why would it be more expensive?
Dylan:	Because like there's more met...square metres
Teacher:	More square metres?
Dylan:	Yeah, like more, like the perimeter is larger than the other ones. Like say they picked, they went for twenty five by four, so the perimeter of that like would be about fifty eight, the other one ten by ten that's only forty
Teacher:	So do you agree with Daniel's point, do you want to then respond to it Karen?
Karen:	We picked the twenty five by four because the other one, the fifty by two we felt was too skinny and if the sheep were to walk around they'll be up real tight in it. The other two that we crossed out that we didn't want to do, well it was really long but really skinny and the other one didn't work because it was twenty five, twenty five.

Classroom Episode 1

Notice that Daniel addresses the girls directly – not through the teacher.

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Daniel:	Yeah, the ones that they were saying were...

Girls want to make sense of what was said but currently cannot.

Daniel introduces a criterion for recommending an enclosure.

Teacher checks understanding

Teacher asks rest of class to listen carefully.

Classroom Episode 1

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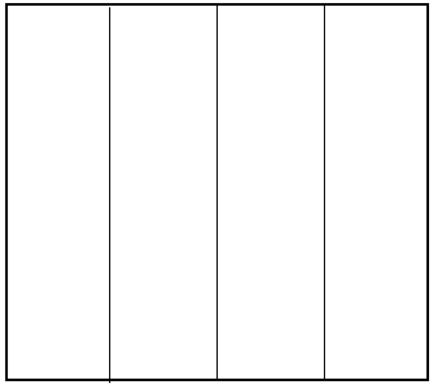
Teacher concerned that other students are not following and checks with reference to one student.

Girls introduce an alternative criterion for recommending an enclosure.

Classroom episode 2

Sixth class
Dividing whole numbers by
unit fractions.

Classroom episode 2



T: **Move it over there to one side; no, no, it’s ok, keep going, keep going, you’re alright.**

S1: Say that’s one, and you’re dividing it by four. [Pause]. It’s how many fours are in. I don’t really see how you draw that.

T: **Have you, have you done that? Is that one divided by four? Is that one divided by a quarter?**

S1: No they’re only asking you for one quarter. Yeah it is actually it is.

S2: Because it’s how many quarters are in one, that’s easy

T: **If you go back to the original, the whole number one, what are you asking yourself, when you’re dividing 72 by nine? You’re saying how many nines are there in 72? Same question here is how many quarters are there in one? So how many quarters are there in one?**

S1: Four

T: **So it is effectively dividing by four isn’t it?**

S1: Yeah, it’s just …

T: **Are you happy with that drawing?**

S1: Yeah, it’s just the answer is all of them, not just one. It’s usually one. Because if you’re quartering it, the answer is one of them, but if you’re ahh, dividing by a quarter it’s all of them, so I just, that’s what I was drawing, the other way.

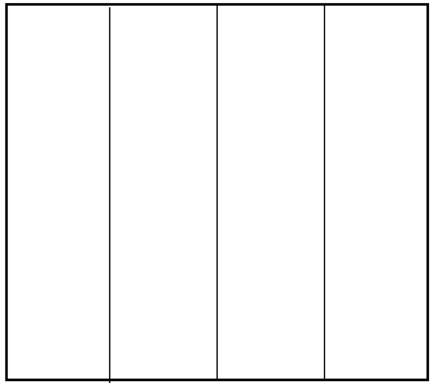
T: **Yeah**

S1: But then that turned out to be wrong

T: **Yeah**

Classroom episode 2

Student's inclination is to try and make sense of problem.



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T: **Yeah**

S1: But then that turned out to be wrong

T: **Yeah**

Teacher connects problem to a related, more familiar, one.

Student resolves confusion by noting that representation has to be interpreted differently to "usual" way.

Make students' ideas public

- Prepare children to listen to one another (and to speak out so they can be heard)
- "Tell us how you figured that out."
- "Who can describe in their own words what ___ did?"
- "Does anyone agree or disagree with how ___ worked it out?"
- "Did anyone do it a different way?"
- "How confident are you in your answer? Why?"
- "What are the benefits of doing it that way/a different way?"

True or False?

- You cannot subtract a larger number from a smaller number.
- Multiplication means repeated addition.
- Multiplying a number increases the size of the number.
- Dividing a number decreases the size of the number.
- The larger the denominator the smaller the fraction.

You cannot subtract a larger number from a smaller number

$$\begin{array}{r} 42 \\ -\ 25 \\ \hline \end{array}$$

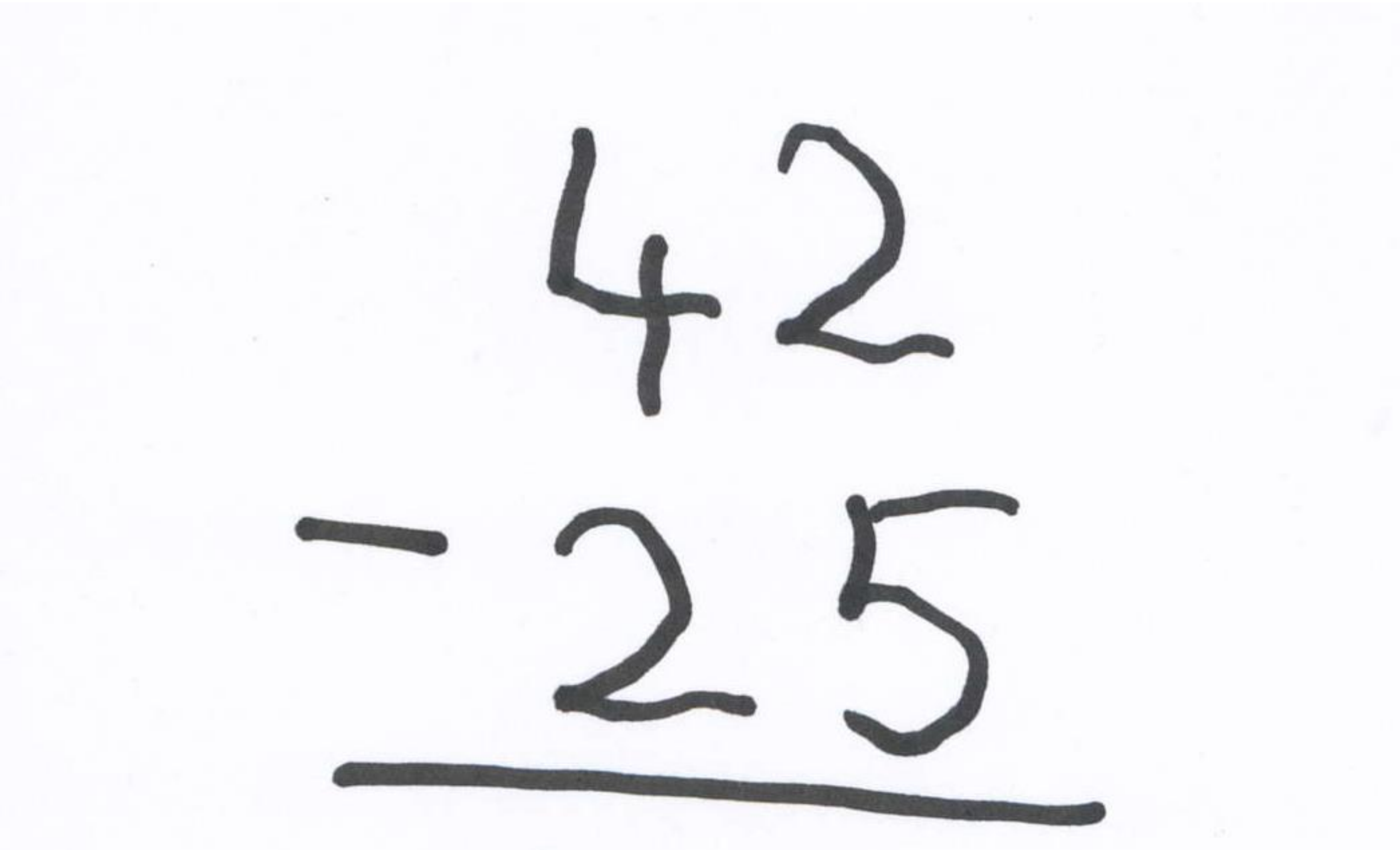
If calculating this in class, it would be common to begin saying, “2 minus 5, I cannot do.”

However, as children get older they learn that 2 minus 5 is -3.

The reason is because implicit in the first expression is that the numbers are restricted to natural or whole numbers whereas the second implies that we have transitioned to the set of integers.

The problem is that we tend to stay silent about the domain of numbers with which we are working which can potentially lead to confusion.

You can subtract a larger number from smaller number by changing your number domain



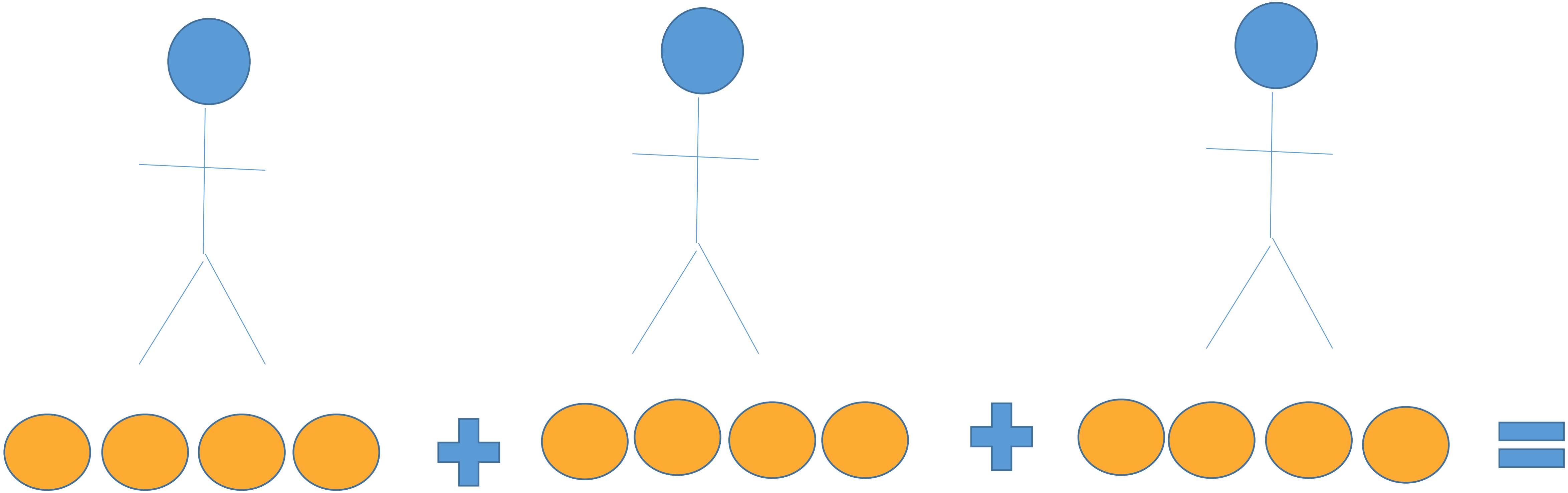
An alternative would be to say that “2 minus 5 we cannot do with the numbers we use in this class.”

Many children will not even notice the subtle difference.

But someone may ask you, are there other numbers? And that is when you can discuss some properties of numbers that they will encounter as they advance through their school years.

Multiplication means repeated addition

- 3 children each have 4 oranges. How many oranges altogether?



4 + 4 + 4 = 12

$3 \times 4 = 12$

Multiplication means repeated addition?

- A piece of elastic can be stretched to 3.3 times its original length. What is the length of a piece 4.2m long when fully stretched?
- I ran ¾ km this morning. My friend ran ½ that distance. How far did my friend run?
- There are 30 children in a class. 2/5 of the children are boys. How many boys are in the class?
- 1cm on a map represents 0.5km. If a distance on the map is 3.6cm, how far is it on the ground?
- What is the area of a rectangle whose dimensions are 3m and 4m?

How would you represent these calculations?

Would repeated addition be helpful for any of them?

Alternatives to presenting multiplication solely as repeated addition

- Repeated addition can help with multiplication calculations initially but the focus remains on units rather than on groups (Clark & Kamii, 1996).
- Embed numbers to be multiplied in contexts, with units, and stories or illustrations, to help students focus on the meaning
- Scale a number by a factor of x. e.g. Scale 2 by a factor of five.

- Use language of double, treble; x times the size of, the weight of, the length of, as much as.
- Size can be increased, reduced or stay the same.
- Link to scale on maps

Sample Multiplication Task 1

- How many ways can you make €1 using only two types of coins (1c, 2c, 5c, 10c, 20c, 50c, 100c)?

Examples:

8 x 5c + 6 x 10c

10 x 2c + 4 x 20c

**You might initially limit the coins that can be chosen*

**You could change the €1 amount*

**Ask children if all possible combinations have been found (and how they can be sure)*

**Prepares children for applying the distributive property of multiplication over addition and the commutative property of addition and multiplication in problems such as 23 x 4 which can be calculated as (4 x 3) + (4 x 20)*

Sample Multiplication Task 2

- Who won the football match if the score was Mayo 1-13 and Down 2-8? By how much did they win?
- (1x3) + (13 x 1) = 3 +13 = 16
- (2x3) + (8 x 1) = 6 + 8 = 14

Multiplying a number increases the size of the number.

- God blessed them, saying to them, “Be fruitful, **multiply,** fill the earth and conquer it.” (Genesis, 1:28).
- True for many numbers, especially natural numbers greater than 1.
- Not true if you multiply a natural number a by 1 ($1 \times a = a$)
- Not true if you multiply a whole number b by 0 ($b \times 0 = 0$)
- Not true if you multiply a positive integer c by a negative integer (e.g. $c \times -1 = -c$)
- Not true if you multiply a rational number d by a proper fraction such as ½ ($d \times ½ = d/2$)
- Not true if you multiply a rational number e by a decimal lower than 1, say 0.5 ($e \times 0.5 = e/2$)

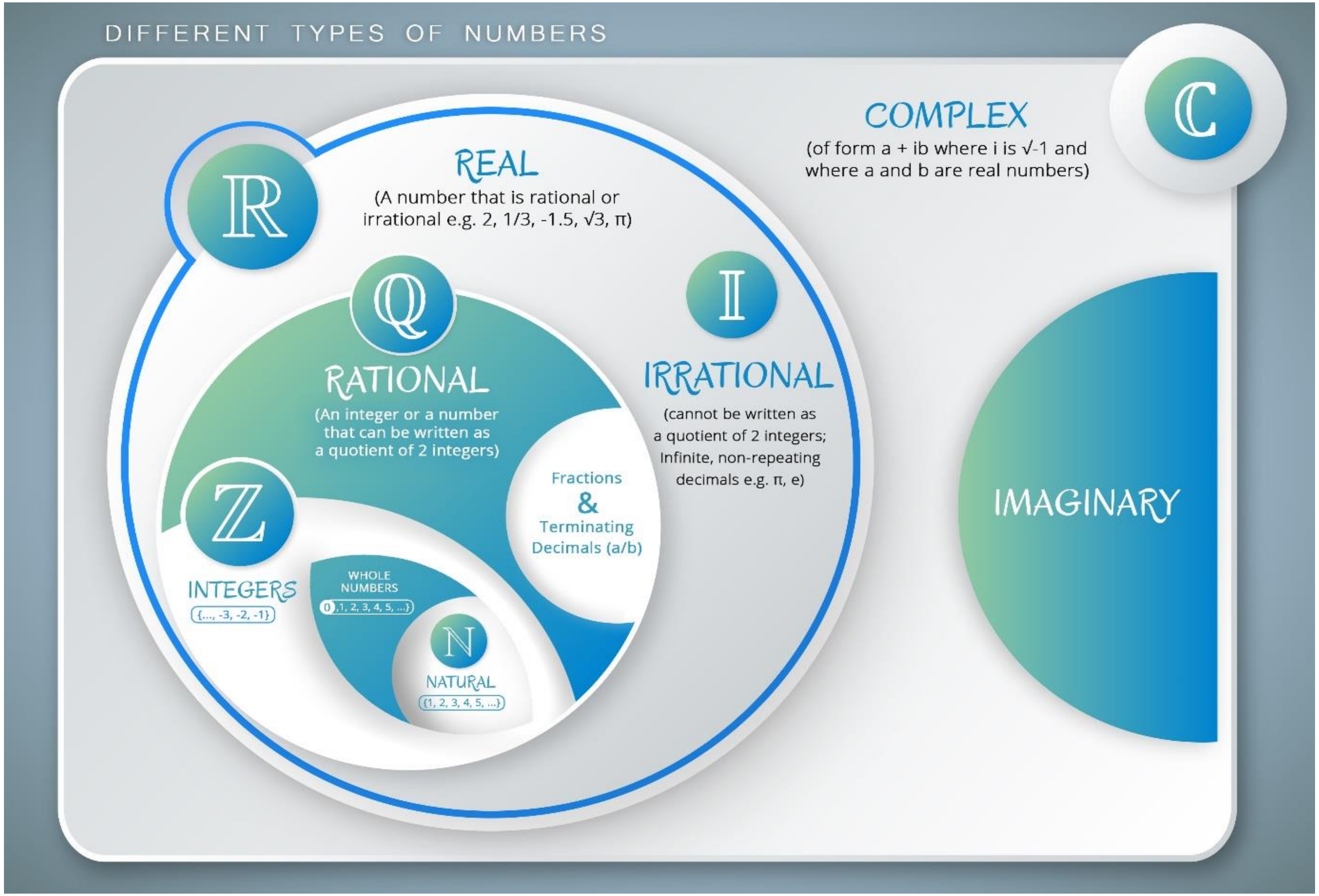
Multiplying a number may increase, decrease or maintain the size of a number

- Multiplier (number being multiplied); Multiplicand (number of groups); Product (multiplier x multiplicand)

N	M	P
"How many base units make one group exactly?"	"How many groups make the product amount exactly?"	"How many base units make the product amount exactly?"
Multiplier	Multiplicand	Product

- The commutative property of multiplication allows us to switch the multiplier and the multiplicand (Haylock, 2010).
- Domains of numbers used in primary school: Natural, whole, integers, rational.
- Would be good to be explicit about the domain of numbers being used to help the children make sense of mathematics.
- Try to surface the misunderstanding that multiplying a number increases the size of the number
- Label the idea as "Mary's conjecture" or "Liam's hypothesis."
- Ask children if they can prove or disprove that the conjecture is always true. You need only one exception to disprove a conjecture.

Number Domains



Dividing a number decreases the size of the number.

- Language of dividing is associated with equal sharing and repeated subtraction.
- True for many numbers, especially natural numbers greater than 1.
- Not true if you divide a natural number x by 1 ($x \div 1 = x$)
- Not true if you divide a negative integer $-y$ by a positive integer (e.g.

$-8 \div 4 = -2$)

- Not true if you divide a rational number z by a proper fraction such as ($12 \div \frac{1}{2} = 24$)
- Not true if you divide a rational number w by a decimal lower than 1, e.g. ($2 \div 0.25 = 8$)
- You cannot divide a number by 0.

Dividing a number decreases the size of the number.

- Use repeated subtraction meaning of division as well as equal sharing
- Embed numbers in real contexts to add meaning.
- E.g. I need ½ apple to make one muffin. How many muffins can I make with 7 apples?
- It takes me 0.2 hours to walk a kilometre. How many kilometres can I walk in 3 hours?
- It takes a scuba diver 4 minutes to dive to 8 metres below sea level. How far does she travel in 1 minute if she travels at a constant speed? $-8 \div 4 = -2$

(However, “real world” examples with negative numbers tend to be contrived)

The larger the denominator, the smaller the fraction

Does this rule make sense with these fractions?

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{9}$ $\frac{1}{3}$ $\frac{1}{15}$

What about with these fractions?

$\frac{3}{4}$ $\frac{3}{2}$ $\frac{3}{9}$ $\frac{3}{3}$ $\frac{3}{15}$

What about?

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{7}{9}$ $\frac{2}{3}$ $\frac{7}{15}$

Students with good knowledge of fraction sizes tend to be more competent in calculating fractions

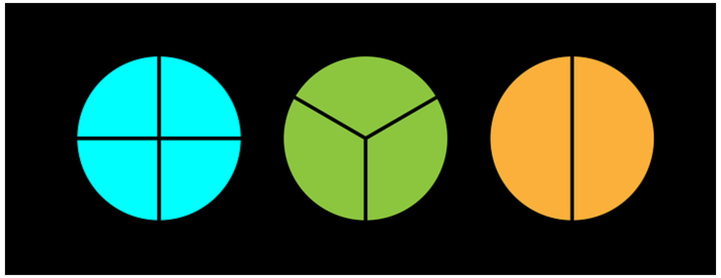
What is a fraction?

Fractions can be

- Part of a whole
- Part of a set
- A line segment
- A point on a number line
- Division (÷)
- An operator e.g. Multiply these numbers by 1/3: 9, 12, 30.
- A ratio (part to part) e.g. girls to boys in a class ($^2/_3$)
- A ratio (part to whole) e.g. girls to children in a class ($^2/_5$)

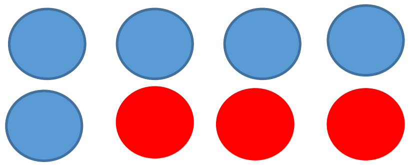
Fraction Models – Part-whole

- Part-whole model of fractions has been prominent in Irish textbooks

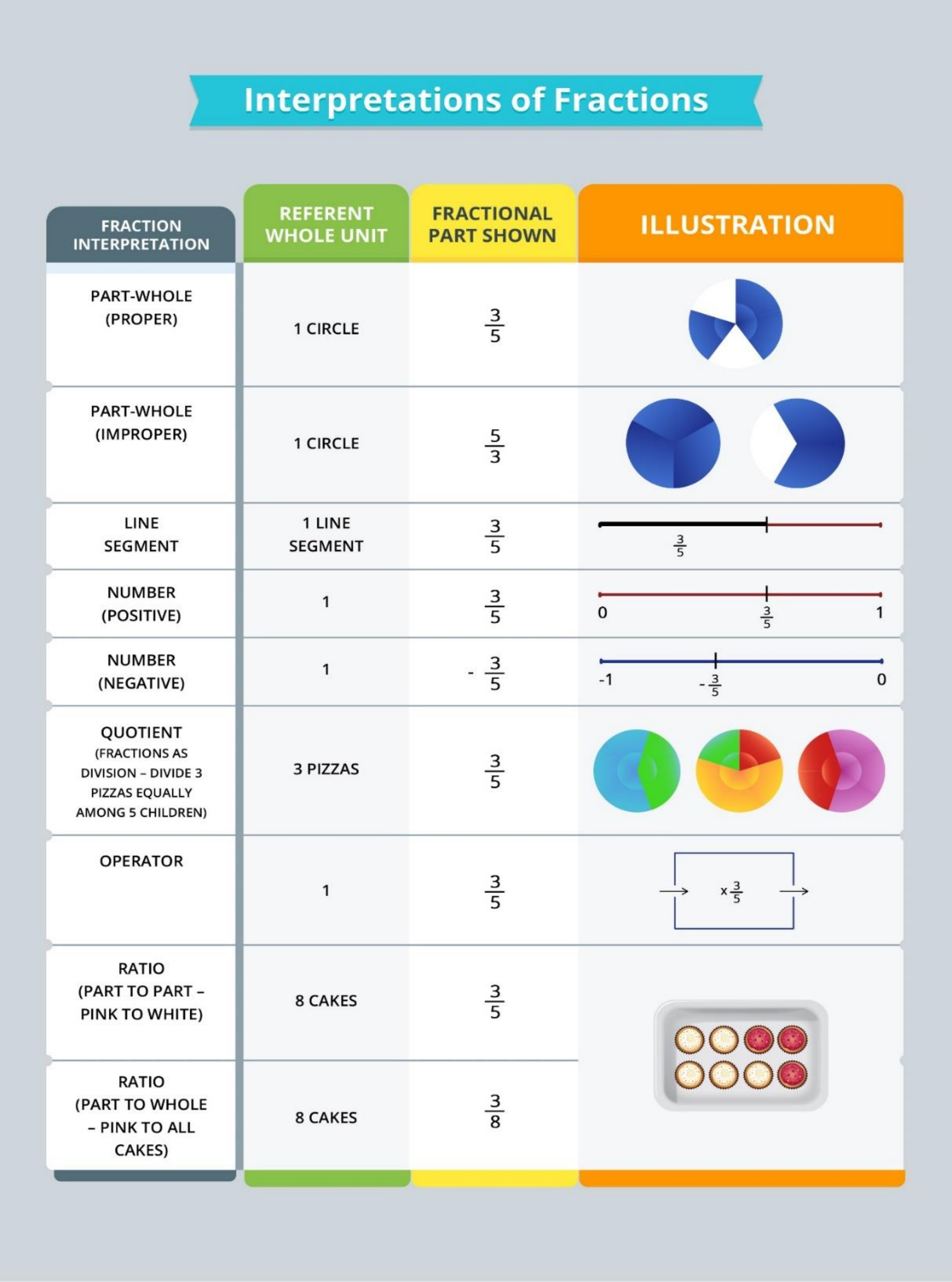


Chocolate
Tart or pie
Pizza

- The set model has also been popular



- Concrete, manipulable, accessible



Which model would you choose to teach

- Equivalent fractions?
- Improper fractions?
- Negative fractions?
- Fractions with large numerators or denominators?
- Connections between fractions and the size/quantity of whole numbers?

Problem Solving Strategies (Polya)

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

Problem Solving Rules

- ~~RUDE (Read, Underline, Draw diagram, Estimate)~~
- ~~ROSE (Read, organise, solve, evaluate)~~
- ~~STAR (Search the word problem – info; Translate the words into an equation or picture – plan; Answer the problem – solve; Review the solution – check.)~~
- ~~LUV2C (Look, Underline, Visualise, Choose numbers, Calculate)~~

- Better to integrate problem solving instruction into regular mathematics teaching rather than treating it as a topic in itself (Fan and Zhu, 2007).

Problem-Solving Heuristics (Fan & Zhu, 2007)

Table 2 A list of specific problem-solving heuristics

	Descriptions
Act it out	Using people or objects to physically show what is exactly described in the problem.
Change your point of view	Approaching a problem from another angle when a previous approach is not effective.
Draw a diagram	Doing a sketch based on the available information to visually represent the problem.
Guess and check	Making a reasonable guess of the answer and then checking the result to see if it works. If necessary, repeating the procedure to find the answer, or at least a close approximation.
Logical reasoning	Demonstrating that if a statement is accepted as true, then other statements could be shown as true based on it.
Look for a pattern	Identifying patterns in the givens based on careful observation of common characteristics, variations, or differences about numbers, shapes, etc. in then problem.
Make a systematic list	Constructing an organized list containing all the possibilities for a givens situation and finally to find the answer.
Make a table	Organizing data into a table and then using the entries in the table to solve the problem.

An informal aid to problem solving – often involving exploration or trial and error

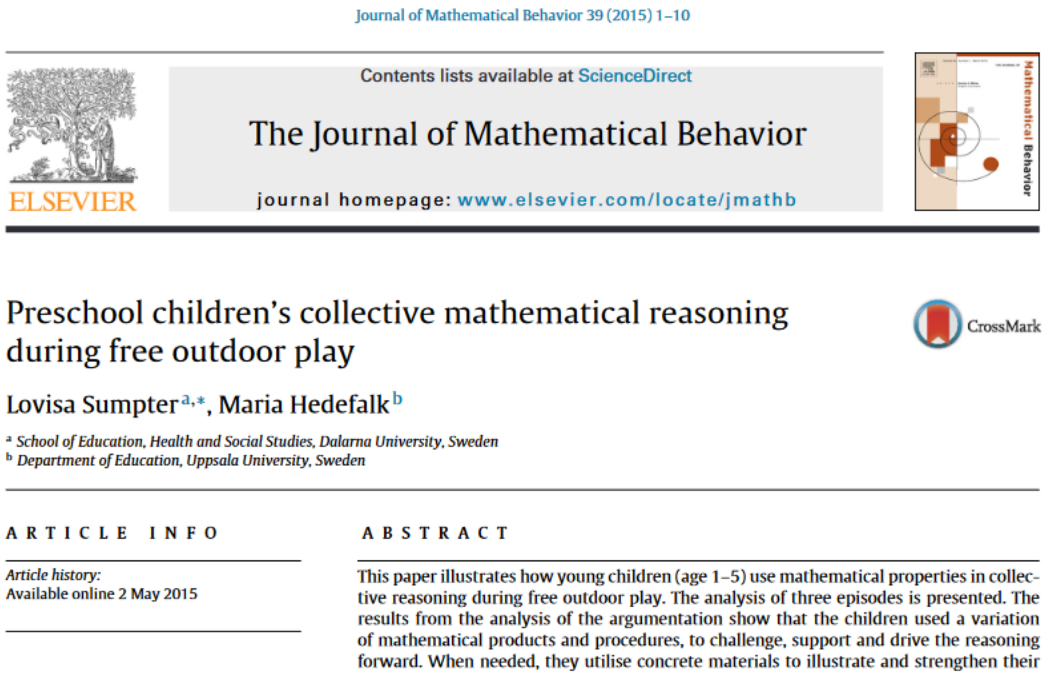
Problem-Solving Heuristics (Fan & Zhu, 2007)

Make suppositions	Making a hypothesis, and then based on the givens and hypothesis, finding out the relationship between the known and unknown, and finally solving the problem.
Restate the problem	Rephrasing the original problem so that the statement of the problem becomes familiar and hence more accessible.
Simplify the problem	Changing the complex numbers or situations in the problem into simpler ones without altering the problem mathematically.
Solve part of the problem	Dividing a problem into several sub-problems, then solving them one by one, and finally solving the original problem.
Think of a related problem	Using methods/results of a related problem, or recalling a related problem, or considering a similar problem solved before in order to solve the problem.
Use a model	Creating visual representations (e.g., using points, lines, or other easy-to-understand symbols) to model the information on quantities or relationships or changes that are involved in the problem.
Use an equation	Using letters as variables to represent unknown quantities in a problem, and establishing and solving equations or inequalities to get the answer.
Use before–after concept	Listing information given before and after action, and observing the change between the two situations (from before to after) to find the solution.
Work backwards	Approaching a problem from its outcomes or solutions backwards to find what conditions they eventually need to meet.

These ideas are all very well but using them will slow down the progress I'll make with the children in maths class.

Would you prefer a builder who said they'd build your house in six months but with a risk of a cracked foundation and a leaky roof or a builder who'd take nine to twelve months but guarantee that the foundation and roof would be sound?

I agree that these ideas could work well in fifth and sixth class but I'm not so sure about infant classes or even first and second.



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Preschool children's collective mathematical reasoning during free outdoor play

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ABSTRACT

This paper illustrates how young children (age 1–5) use mathematical properties in collective reasoning during free outdoor play. The analysis of three episodes is presented. The results from the analysis of the argumentation show that the children used a variation of mathematical products and procedures, to challenge, support and drive the reasoning forward. When needed, they utilise concrete materials to illustrate and strengthen their

Even young children (aged 1 to 5) have been found to reason mathematically in play settings, using oral language skills and concrete objects. They questioned, challenged and supported each others' ideas about number of siblings, and height. Even without direct teacher intervention, they could create collective knowledge as co-teachers and co-learners. (Sumpter and Hedefalk, 2015).

1. Instead of recording if every child can use the same method, record if every child can reliably calculate a given operation (e.g. division can be solved using equal sharing or repeated subtraction).
2. If children are forced into particular methods, their learning will likely remain more fragile (and based on memorisation) than if they can choose and refine their own way of understanding (based on making sense).
3. In standardised tests, it is the answer rather than the method that is judged.

It’s much easier to teach the class and keep track of children’s progress in mathematics when every child uses the same approach to solving problems.

1. You could indirectly admit to the children that you need some convincing. For example, say “Supposing someone cannot understand what you are saying, how would you explain it to them in simple terms/in a different way?”
2. You could ask the children to write down a question they have about the day’s work or something they now think is true that they previously did not know or an explanation for something discussed in class. Then read over them at your own pace after class.

I’m not that confident in my own mathematics. If I let the children share their own ideas, I might get confused myself and lose face in front of the children.

To Sum Up

- When teaching mathematics, ask “how do I know if this material is making sense to students?”
- Work together with students to see how mathematics makes sense (or why sometimes it seems not to)
- Think about how to keep ideas consistent across the curriculum – subtracting numbers; introducing multiplication; how multiplying and dividing change numbers; multiple meanings of fractions.
- Make students independent of you by asking them “how did you figure that out?” when they give an answer (regardless of whether their answer is correct or incorrect).

Ask questions such as…

How confident are you in that answer/solution?

Why?

How did you get that?

Esti

Could it

ay?

EXPLAIN

Compare that to…

Tell me about…

t into your own words

does that connect with…?

Descr

Do you agree or disagree?

How can you be sure?

…and genuinely engage with the response.

Ask questions such as…

How confident are you in that answer/solution?

Why?

HOW DID YOU GET THAT?

Estimate

Could it be done another way?

EXPLAIN

Compare that to…

Tell me about…

Put into your own words

How does that connect with…?

Describe

Do you agree or disagree?

How can you be sure?

My “money-back” guarantee!

- Thank you for attending the webinar.
- From tomorrow, take one step to help your students make sense of the mathematics they are learning.
- Let me know how you get on.
- If you encounter problems, let me know and if I can suggest a fix, I’ll do so.
- Please write to me: sean.delaney@mie.ie.
- Slides: www.seandelaney.com.

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